

KENDRIYA VIDYALAYA SANGATHAN, NEW DELHI

केन्द्रीय विद्यालय संगठन ,नई दिल्ली

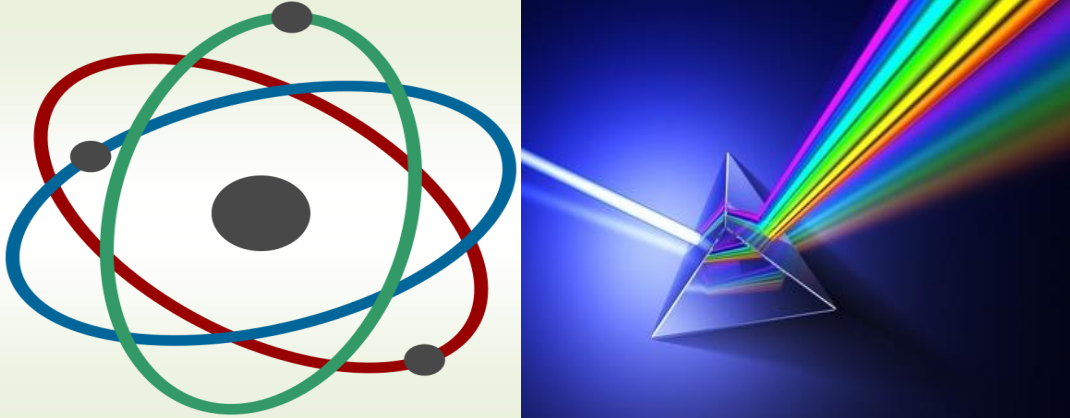


स्नातकोत्तर शिक्षक (भौतिक) हेतु भौतिकि ओलम्पियाड पर
3 दिवसीय कार्यशाला
रिपोर्ट – कम –मेनुअल

REPORT-CUM-MANUAL

3 DAY WORKSHOP ON PHYSICS OLYMPIAD FOR PGT (PHYSICS)

FROM 09/08/2017 TO 11/08/2017



आंचलिक शिक्षा एवं प्रशिक्षण संस्थान मुम्बई

ZONAL INSTITUTE OF EDUCATION & TRAINING, MUMBAI

NCH Colony Kanjurmarg (West) Mumbai-44078

संरक्षक

PATRONS

श्री संतोष कुमार मल्ल
माननीय आयुक्त
के० वि० संगठन, नई दिल्ली.
Mr.Santosh Kumar Mall, IAS
Hon'ble Commissioner
KVS New Delhi



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KVS New Delhi



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के० वि० संगठन, नई .दिल्ली
Dr. Shachi kant
Joint Commissioner (Trg.)
KVS New Delhi



डॉ वी विजयलक्ष्मी
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डॉ. ई.प्रभाकर
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Joint Commissioner (Admin)
KVS New Delhi



आभारोक्ति
ACKNOWLEDGMENTS

सुश्री ऊषा अश्वत्थ अय्यर
उपायुक्त एवम निदेशक ऑमुम्बई .सं.प्र.शि.
पाठ्यक्रम निदेशक

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DEPUTY COMMISSIONER & DIRECTOR
COURSE DIRECTOR
ZIET MUMBAI

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प्रशिक्षण सहायक (भौतिकि)
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TRAINING ASSOCIATE (PHYSICS)
RESOURCE PERSON
ZIET, MUMBAI

श्री यूजिन डी लीन
Mr. Eugin D Leen,
Training Associate (English)
COURSE COORDINATOR
ZIET, MUMBAI

SUPPORTED BY FACULTY & STAFF ZIET MUMBAI

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प्रशिक्षण सहायक,वाणिज्य

श्री प्रभाकरजिल्ला,
वरिष्ठ सचिवालयसहायक
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Secretariate

Mr. S.K. Singhal
Training Associate (Commerce)

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प्रशिक्षण सहायक प्राथमिक
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Training Associate (Primary)

श्रीमती जोयस जे.पी .,स्टेनो
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Sr.Stenographer

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प्रशिक्षण सहायक (प्राथमिक)
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Sub staff

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पुस्तकालयाध्यक्ष
Mrs. Kanta Bara
Librarian

श्री गोपीराम बाल्मीक,
सबस्टाफ-
Mr. Gopiram valmiki,
Sub staff

MESSAGE



This was the first time that ZIET Mumbai collaborated with Homi Bhabha Centre for Science Education and designed a workshop to make teachers aware of the 'Physics Olympiad'. It was a three-day workshop for PGTs of Physics. Experts from HBCSE, Mumbai took sessions on various topics and also helped the participants prepare material which can be used by the students.

It was a novel opportunity for the teachers to work in an area not directly related to the syllabus. It also exposed the teachers to setting of questions for a challenging competition like the Olympiads.

KVS wants its students to take part in as many different activities as possible- be it sports, cultural, social or Olympiads. Olympiads promote higher level thinking skills and will test the true mettle of the student.

I hope the materials prepared at ZIET Mumbai as well as the material prepared by our PGT Physics, Mr M G Reddy will be of use to the students.

USHA ASWATH IYER

DIRECTOR, KVS ZIET MUMBAI

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**3-DAY WORKSHOP FOR PGT (PHYSICS) ON PHYSICS OLYMPIAD
FROM 09.08.2017 TO 11.08.2017, VENUE:- ZIET, MUMBAI**

DATE	0900-1045		1100-1.00 P.M	1 P.M-2 P.M	2.00P.M--3.45 P.M		1600-1730	
09.08.2017	Objectives of Workshop, Details regarding different types of Olympiad	Tea Break	Olympiads- an Over View	Lunch Break	Physics Olympiad problems	Tea Break	Preparation of Olympiad questions with Answers -Group Work	
	Mr Gopala Reddy TA(Physics)		Prof. Anwesh Mujumdar , HBCSE, TIFR, National coordinator		Dr.Pravin Pathak, HBCSE, TIFR, MUMBAI		Mr Gopala Reddy TA(Physics)	
10.08.2017	Preparation of Olympiad questions with Answers -Group Work		Practical Session- Experiments		Practical session- Experiments		Mr. Shirish, Scientific officer, HBCSE	Practical session- Experiments
	Mr. M.G. REDDY,TA(PHY)		Mr. Shirish, Scientific officer, HBCSE		Mr. Shirish, Scientific officer officer HBCSE			Mr. Shirish, Scientific officer, HBCSE
11.08.2017	Preparation of Olympiad questions with Answers -Group Work		Preparation of Olympiad questions with Answers -Group Work		Presentation of Group work and discussion		Mr. M.G. REDDY,TA(PHY)	Presentation of Group work and discussion
	Mr. M.G. REDDY,TA(PHY)		Mr. M.G. REDDY,TA(PHY)		Mr. M.G. REDDY,TA(PHY)			Mr. M.G. REDDY,TA(PHY)

LIST OF PARTICIPANTS ATTENDED FOR 3 DAY WORKSHOP ON “PHYSICS OLYMPIAD

S. No	Name of the participants	Name of the KV	Region
1	Sh. Prakash Singh	No.1 AFS Suratgarh	JAIPUR
2	Sh.S.D.Pareek	BSF Anupgarh	JAIPUR
3	Sh.Ajeet Singh Monga	No.1 Army Jodhpur	JAIPUR
4	Sh.Deepak Jangid	BSF Pokaran	JAIPUR
5	Sh.M.K.Sharma	Bharatpur	JAIPUR
6	Sh.C.Pandya	No.1 Udaipur	JAIPUR
7	Sh.Shambhy Prakash Bairwa	Dungarpur	JAIPUR
8	Sh.Sandeep Kumar Koli	No.2 Ajmer	JAIPUR
9	Sh.Rajesh Gupta	No.2 Jaipur	JAIPUR
10	Sh.Ratan Pal	Churu	JAIPUR
11	Sh.Amit Kumar Paliwal	Sikar	JAIPUR
12	Mr. Alok	NAD Karanja	MUMBAI
13	Mr. C. R. Ramteke	Kamptee	MUMBAI
14	Mr. Kamlesh Dharne	VSN Nagpur	MUMBAI
15	Mr. Birender Singh	No. 1 Vasco-Da-Gama	MUMBAI
16	Mr. G. G. Bhagat	Yavatmal	MUMBAI
17	Mrs. Varsha Kadam	No. 1 Ahmednagar	MUMBAI
18	Mr. Sunil Jadhav	Aurangabad Cantt	MUMBAI
19	Mr. Dhananjay Parkar	No. 1 Devlali	MUMBAI
20	Mr Rajesh Kumar	BEG , Pune	MUMBAI
21	Mr. Prem Kumar Bind	Okha	AHMEDABAD
22	Sh. Akhilesh Suryavanshi	Kv No. 2 Army Bhuj	AHMEDABAD
23	Mr. P K Sah	VV Nagar	AHMEDABAD
24	sh. C. M. Sharma	ONGC Ankleshwar	AHMEDABAD
25	Sh. Teekam Chand Agrawal	KV No.1 Sec30 Gandhinagar	AHMEDABAD
26	Sh. A.Jebin Joel	AECS NO.4	MUMBAI
27	Smt. Lekha Pillai	AECS NO.4	MUMBAI

3 day workshop for P.G.T(Physics) on "PHYSICS OLYMPIAD"

From 09/08/2017 to 11/08/2017

FORMATION OF GROUPS AND DISTRIBUTION OF GROUP WORK

GROUP 1: C.V. RAMAN

S.NO	NAME OF TEACHER	NAME OF KV	REGION	NAME OF THE UNIT
1	Sh. Prakash Singh	No.1 AFS Suratgarh	Jaipur	UNIT-I- ELECTROSTATICS
2	Sh.S.D.Pareek	BSF Anupgarh	Jaipur	UNIT-II- CURRENT ELECTRICITY
3	Smt. Lekha Pillai	AECS NO.4	Mumbai	UNIT-I- ELECTROSTATICS
4	Sh. A.Jebin Joel	AECS NO.4	Mumbai	UNIT-II- CURRENT ELECTRICITY

GROUP 2: NEWTON

S.NO	NAME OF TEACHER	NAME OF KV	REGION	NAME OF THE UNIT
1	Sh.Ajeet Singh Monga	No.1 Army Jodhpur	Jaipur	UNIT-III-MAGNETIC EFFECTS OF CURRENTS& MAGNETISM
2	Sh.Deepak Jangid	BSF Pokaran	Jaipur	UNIT-IV- E.M.I.&A.C.
3	Mr. Alok	NAD Karanja	Mumbai	UNIT-III-MAGNETIC EFFECTS OF CURRENTS & MAGNETISM
4	Mr. C. R. Ramteke	Kamptee	Mumbai	UNIT-IV- E.M.I.&A.C

GROUP 3: -JAGDISH CHANDRA BOSE

S.NO	NAME OF TEACHER	NAME OF KV	REGION	NAME OF THE UNIT
1	Mr. Kamlesh Dharne	VSN Nagpur	Mumbai	UNIT-VI- OPTICS
2	Sh.Shambhy Prakash Bairwa	Dungarpur	Jaipur	UNIT-IX- ELECTRONIC DEVICES
3	Mr. G. G. Bhagat	Yavatmal	Mumbai	UNIT-IX- ELECTRONIC DEVICES

GROUP4: - HOMI BHABA

S.NO	NAME OF TEACHER	NAME OF KV	REGION	NAME OF THE UNIT
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1	Sh.C.Pandya	No.1 Udaipur	Jaipur	UNIT-VIII- ATOMS &NUCLIE, UNIT-V- E.M. WAVES
2	Mr. Birender Singh	No. 1 Vasco-Da-Gama	Mumbai	UNIT-VII- DUAL NATURE OF MATTER & RADIATION, UNIT-X-COMMUNICATION
3	Mr. Rajesh Kumar	BEG Pune	Mumbai	UNIT-VIII- ATOMS &NUCLIE UNIT-V- E.M. WAVES

GROUP:- 5:- ALBERT EINSTIEN

S.NO	NAME OF TEACHER	NAME OF KV	REGION	NAME OF THE UNIT
1	Sh. Teekam Chand Agrawal	KV No.1 Sec30 Gandhinagar	AHMEDABAD	UNIT-II – KINEMATICS
2	Sh.Sandeep Kumar Koli	No.2 Ajmer	JAIPUR	UNIT-II – KINEMATICS
3	Mrs. Varsha Kadam	No. 1 Ahmednagar	Mumbai	Unit-II – KINEMATICS
4	Sh.Amit Kumar Paliwal	Sikar	Jaipur	UNIT- VI – GRAVITATION

GROUP 6: - VIKRAM SARA BHAI

S.NO	NAME OF TEACHER	NAME OF KV	REGION	NAME OF THE UNIT
1	Sh. Akhilesh Suryavanshi	Kv No. 2 Army Bhuj	AHMEDABAD	UNIT-IX- KINETIC THEORY OF GASES UNIT-I- PHYSICAL WORLD AND MEASUREMENT
2	Mr. Prem Kumar Bind	Okha	AHMEDABAD	UNIT-X- OSCILLATIONS AND WAVES
3	Mr. P K Sah	V V Nagar	AHMEDABAD	UNIT-VIII- THERMODYNAMICS
4	sh. C. M. Sharma	ONGC Ankleshwar	AHMEDABAD	UNIT-VII- PROPERTIES OF BULK MATTER

GROUP 7: MICHAEL FARADAY

S.NO	NAME OF TEACHER	NAME OF KV	REGION	NAME OF THE UNIT

1	Sh.Rajesh Gupta	No.2 Jaipur	JAIPUR	UNIT – III – LAWS OF MOTION
2	Mr. Sunil Jadhav	Aurangabad Cantt	Mumbai	Unit – III – LAWS OF MOTION
3	Sh.Ratan Pal	Churu	Jaipur	UNIT- IV- WORK, ENERGY AND POWER
4	Mr. Dhananjay Parkar	No. 1 Devlali	Mumbai	Unit- IV- WORK, ENERGY AND POWER
5	Sh.M.K.Sharma	Bharatpur	Jaipur	Unit- V – MOTION OF SYSTEM OF PARTICLES

REPORT OF DAY -1 OF WORKSHOP ON PHYSICS OLYMPIAD

The 3 day workshop on Physics Olympiad started with prayer by all the participating teachers from three regions Jaipur, Ahmedabad and Mumbai of KVS and the participants from Atomic Energy school. This was followed by the self-introduction by all the participants of workshop.

Then a short introductory speech was given by Resource person, Mr. M G Reddy Training Associate ZIET Mumbai. He briefed the participants about the Aims & Objectives of the workshop and the need of the hour to participate in Olympiads. This was followed by a key note address by the Course Director Ms. Usha Ashwath Iyer, Director & Deputy Commissioner ZIET Mumbai which inspired the participants to be self-motivated. She also encouraged the teacher participants to enjoy the stay at ZIET Mumbai.

After inauguration, first session was taken by Mr. M G Reddy, Training Associate ZIET Mumbai explaining about different types of Olympiads such as IAPT, KVPY, SOF, NCSC and Junior Science Olympiads. He also explained about different stages of Olympiads and explained in detail about Do's & Don'ts. He presented multiple choice type questions prepared by him covering all the topics of class XI & XII.

Post tea break session was delivered by Mr. Anwesh Majumdar, Associate Professor Homi Bhabha Centre for Science Education (HBCSE) & National Coordinator National Science Olympiads. He gave a very vivid picture of the Olympiad programme. He elaborated the selection process, the preparation for the conduct of test and the rigorous training given to the selected candidates.

This was followed by the interactive discussion on the problems of electricity and magnetism. The lecture was enjoyed by the participants as it removed various misconceptions of these topics. The discussion was so interesting that it continued even after the lunch break also.

The afternoon session on Graphs, Diagrams and Physics was delivered by Mr. Praveen Pathak of HBCSE. He discussed the problems of kinematics and rotational motion and cleared the various concepts of these topics.

Finally the day ended with the formulation of Groups for preparation of material of Physics Olympiad. The participants were allotted topics to prepare the study material.

DAY -2

Report of the day date 10/08/2017 By Birender Singh, PGT (PHY), KV No1, Vasco, Goa
Proceedings of the second day started with morning assembly by Jaipur Region teachers.

After the morning assembly much awaited moment came when we all lead by Mr. M .G Reddy, Training Associate, Physics, ZIET, and Mumbai boarded a bus and left for Homi Bhabha Centre for science education. Many participants including me were thrilled by the name of HBCSE since last night.

Calm and cool atmosphere of HBCSE welcomed us all. From our entry into the campus to the exit everything was systematically planned. Without wasting any time we were taken to Olympiad section of HBCSE where we were made comfortable by Dr. Shirish Pathare and his associates Ms. Shrevani, Mr. Pradeep and Mr. Vikas.

Session was started by Dr. Shirish with difference between demonstration and experimentation, knowledge about accuracy and precision, error analysis, graph plotting and its importance.

Shirish sir clearly demonstrated how to linearize the relation $S = ut + at^2/2$ and how to find initial velocity using slope. The session was interactive and thoroughly enjoyed by all.

Then we break for tea and assembled after 10 minutes. Now it was the time for us to become students and perform the experiments and find out the desired result as we expect from our students.

Two activities were allotted to us:-

- (1) To find density of steel ball
- (2) To find Coulomb's force between magnets.

We were divided in two groups. One group was assisted by Ms. Shrevani and Mr. Vikas while the other group assisted by Mr. Pradeep. In the beginning it seemed to be very easy for us but soon we realized that these activities were different from the traditional one. At one pm. We had lunch after the lunch break all rushed towards the lab for completing their assigned activities. Enthusiasm and interest was evident on the faces of all the participants. At 3 pm. We switched on to the next activity and completed both activities successfully under the able guidance of Dr., Sirish and his team. At the last there was question answer session where we discussed a lot about the courses available to us. At about 5 pm we left the venue and reported back to ZIET Mumbai.

Indeed it was a learning experience for all of us and in such a small duration we learnt lot of skills which will be helpful for us in future.

In the end on behalf of all the participants I am thankful to course director Usha Aswath Iyer, DC, ZIET, Mumbai and the members of ZIET, Mumbai without their efforts and planning this day was not possible.

DAY -3

“Improvement is achieved by the ripple effect of a few simple changes in approach, attitude or habit”
Dale Ludwig

On 11th August 2017, morning 9am, the concluding day of the Olympiad work shop started with a Memorable assembly program conducted by the participants from Mumbai region. After prayer, Thought for the day was given by Shri Kamlesh Dharne from K V, VSN Nagpur . Shri Sunil Hiranman Jadhav from K V Aurangabad made a professional and thorough presentation of the day's news headlines. A comprehensive report on the previous day's program held in the HBCSE was presented by Shri Birender Singh of K V No 1 Vasco, Goa.

This was followed by a brief but very enriching talk given by Usha Aswath Iyer, Director ZIET , Mumbai. Starting from the assembly program conducted by Mumbai region, madam touched various minute areas of teaching learning process and gave valuable tips and suggestions. This included innovative methods to make class room teaching more activity oriented and lively with students of 9th and 11th standards. She also suggested to document these activity sessions and to make a report of the same during science day celebrations every year. Madam concluded by urging the participants to devise own techniques to overcome difficulties rather than to lament on them.

The entire team then gathered outside for a group photograph session with the director. On return, the participating teachers were divided in to groups by Shri M Gopala Reddy, Resource person of the program, for preparation of Olympiad Questions from 11th and 12th standard syllabus. The team parted for lunch on completion of this work.

Post lunch, presentation and discussion of the group work was conducted after which, a brief tea break followed.

The concluding part of this very enriching three day program was valedictory function in which valuable feedbacks were given by representatives from each region. The course director Usha Aswath Iyer distributed certificates to all the participants. The program came to a successful conclusion with a parting speech of blessings from the director.

UNIT I – PHYSICAL WORLD AND MEASUREMENT

GIST AND FORMULAE OF PHYSICAL WORLD AND MEASUREMENT

Name Country of Origin	Major contribution/discovery
Archimedes Greece	Principle of buoyancy; Principle of the lever
Galileo Galilei Italy	Law of inertia
Christiaan Huygens Holland	Wave theory of light
Isaac Newton U.K. Universal Reflecting telescope	law of gravitation; Laws of motion;
Michael Faraday U.K.	Laws of electromagnetic induction
James Clerk Maxwell U.K. electromagnetic wave	Electromagnetic theory; Light-an
Heinrich Rudolf Hertz Germany	Generation of electromagnetic waves
Aeroplane	Bernoulli's principle in fluid dynamics
Steam engine	Laws of thermodynamics
Nuclear reactor	Controlled nuclear fission
Radio and Television Generation	propagation and detection of electromagnetic waves
Computers	Digital logic
Lasers	Light amplification by stimulated emission of radiation
Production of ultra high magnetic fields	Superconductivity
Rocket propulsion	Newton's laws of motion

Electric generator Faraday's laws of electromagnetic induction

Hydroelectric power Conversion of gravitational potential energy into electrical energy

Particle accelerators Motion of charged particles in electromagnetic fields

Sonar Reflection of ultrasonic waves

Optical fibres Total internal reflection of light Non-reflecting coatings Thin film optical interference

Electron microscope Wave nature of electrons

Photocell Photoelectric effect

Fusion test reactor (Tokamak) Magnetic confinement of plasma

Giant Metrewave Radio Detection of cosmic radio waves

Telescope (GMRT)

Bose-Einstein condensate Trapping and cooling of atoms by laser beams and magnetic fields.

The comparison of any physical quantity with its standard unit is called **measurement**.

Physical Quantities

All the quantities in terms of which laws of physics are described, and whose measurement is necessary are called physical quantities.

Units

- A definite amount of a physical quantity is taken as its standard unit.
- The standard unit should be easily reproducible, internationally accepted.

Fundamental Units

Those physical quantities which are independent to each other are called fundamental quantities and their units are called fundamental units.

S.No. Fundamental Quantities Fundamental Units Symbol

1. Length	metre
m	
2. Mass	kilogram
kg	
3. Time	second
S	
4. Temperature	kelvin
kg	
5 Electric current	ampere
A	
6 Luminous intensity	candela
cd	
7 Amount of substance	mole

mol

Supplementary Fundamental Units

Radian and steradian are two supplementary fundamental units. It measures plane angle and solid angle respectively.

S.No. Supplementary Fundamental Quantities Supplementary Unit Symbol

1 Plane angle	radian	rad
2 Solid angle	steradian	Sr

Derived Units

Those physical quantities which are derived from fundamental quantities are called derived quantities and their units are called derived units.e.g., velocity, acceleration, force, work etc.

Definitions of Fundamental Units

The seven fundamental units of SI have been defined as under.

- 1 kilogram** A cylindrical prototype mass made of platinum and iridium alloys of height 39 mm and diameter 39 mm. It is mass of 5.0188×10^{25} atoms of carbon-12.
- 1 metre** 1 metre is the distance that contains 1650763.73 wavelength of orange-red light of Kr-86.
- 1 second** 1 second is the time in which cesium atom vibrates 9192631770 times in an atomic clock.
- 1 kelvin** 1 kelvin is the $(1/273.16)$ part of the thermodynamics temperature of the triple point of water.
- 1 candela** 1 candela is $(1/60)$ luminous intensity of an ideal source by an area of cm^2 when source is at melting point of platinum (1760°C).
- 1 ampere** 1 ampere is the electric current which it maintained in two straight parallel conductor of infinite length and of negligible cross-section area placed one metre apart in vacuum will produce between them a force 2×10^{-7} N per metre length.
- 1 mole** 1 mole is the amount of substance of a system which contains a many elementary entities (may be atoms, molecules, ions, electrons or group of particles, as this and atoms in 0.012 kg of carbon isotope ^{12}C).

Systems of Units

A system of units is the complete set of units, both fundamental and derived, for all kinds of physical quantities. The common system of units which is used in mechanics are given below:

- CGS System** In this system, the unit of length is centimetre, the unit of mass is gram and the unit of time is second.
- FPS System** In this system, the unit of length is foot, the unit of mass is pound and the unit of time is second.
- MKS System** In this system, the unit of length is metre, the unit of mass is kilogram and the unit of time is second.
- SI System** This system contain seven fundamental units and two supplementary Fundamental units.

Relationship between Some Mechanical SI Unit and Commonly Used Units

S.No.	Physical Quantity	Unit
1	Length	(a) 1 micrometre = 10^{-6} m (b) 1 angstrom = 10^{-10} m
2	Mass	(a) 1 metric ton = 10^3 kg (b) 1 pound = 0.4537 kg (c) 1 amu = 1.66×10^{-23} kg
3	Volume	

$$1 \text{ litre} = 10^{-3} \text{ m}^3$$

4. Force
 (a) 1 dyne = 10^{-5} N
 (b) 1 kgf = 9.81 N
5. Pressure
 (a) 1 kgf/m² = 9.81 N/m²
 (b) 1 mm of Hg = 133 N/m²
 (c) 1 pascal = 1 N/m²
 (d) 1 atmosphere pressure = 76 cm of Hg = 1.01 x 10⁵ pascal
6. Work and energy
 (a) 1 erg = 10^{-7} J
 (b) 1 kgf-m = 9.81 J
 (c) 1 kWh = 3.6 x 10⁶ J
 (d) 1 eV = 1.6 x 10⁻¹⁹ J
7. Power
 (d) 1 kgf- ms⁻¹ = 9.81 W
 1 horse power = 746 W

Some Practical Units

1. 1 fermi = 10^{-15} m
2. 1 X-ray unit = 10^{-13} m
3. 1 astronomical unit = 1.49×10^{11} m (average distance between sun and earth)
4. 1 light year = 9.46×10^{15} m
5. 1 parsec = 3.08×10^{16} m = 3.26 light year

Some Approximate Masses

Object Kilogram

Our galaxy 2×10^{41}

Sun 2×10^{30}

Moon 7×10^{22}

Asteroid Eros 5×10^{15}

Dimensions

Dimensions of any physical quantity are those powers which are raised on fundamental units to express its unit. The expression which shows how and which of the base quantities represent the dimensions of a physical quantity, is called the dimensional formula.

Dimensional Formula of Some Physical Quantities

Physical Quantity	Dimensional Formula	MKS Unit
2 Volume	$[L^3]$	metre ³
3 Velocity	$[LT^{-1}]$	ms ⁻¹
4 Acceleration	$[LT^{-2}]$	ms ⁻²
5 Force	$[MLT^{-2}]$	newton (N)
6 Work or energy	$[ML^2T^{-2}]$	joule (J)

7 Power	$[ML^2T^{-3}]$	J s-1 or watt
8 Pressure or stress	$[ML^{-1}T^{-2}]$	Nm-2
	$[MLT^{-1}]$	kg ms-1
9 Linear momentum or Impulse		
10 Density	$[ML^{-3}]$	kg m-3
11 Strain Dimensionless		Unitless
12 Modulus of elasticity	$[ML^{-1}T^{-2}]$	Nm-2
13 Surface tension	$[MT^{-2}]$	Nm-1
14 Velocity gradient	T^{-1}	second-1
15 Coefficient of velocity	$[ML^{-1}T^{-1}]$	kg m-1s-1
16 Gravitational constant	$[M^{-1}L^3T^{-2}]$	Nm ² /kg ²
17 Moment of inertia	$[ML^2]$	kg m ²
18 Angular velocity	$[T^{-1}]$	rad/s
19 Angular acceleration	$[T^{-2}]$	rad/S ²
20 Angular momentum	$[ML^2T^{-1}]$	kg m ² S-1
21 Specific heat	$L^2T^{-2} \theta^{-1}$	Kcal kg-1K-1
22 Latent heat	$[L^2T^{-2}]$	kcal/kg
23 Planck's constant	ML^2T^{-1}	J-s
24 Universal gas constant	$[ML^2T^{-2}\theta^{-1}]$	J/mol-K

Homogeneity Principle

If the dimensions of left hand side of an equation are equal to the dimensions of right hand side of the equation, then the equation is dimensionally correct. This is known as **homogeneity principle**.

Mathematically [LHS] = [RHS]

Applications of Dimensions

1. To check the accuracy of physical equations.
2. To change a physical quantity from one system of units to another system of units.
3. To obtain a relation between different physical quantities.

Significant Figures

In the measured value of a physical quantity, the number of digits about the correctness of which we are sure plus the next doubtful digit, are called the significant figures.

Rules for Finding Significant Figures

1. All non-zeros digits are significant figures, e.g., 4362 m has 4 significant figures.
2. All zeros occurring between non-zero digits are significant figures, e.g., 1005 has 4 significant figures.
3. All zeros to the right of the last non-zero digit are not significant, e.g., 6250 has only 3 significant figures.
4. In a digit less than one, all zeros to the right of the decimal point and to the left of a non-zero digit are not significant, e.g., 0.00325 has only 3 significant figures.
5. All zeros to the right of a non-zero digit in the decimal part are significant, e.g., 1.4750 has 5 significant figures.

Significant Figures in Algebraic Operations

(i) In Addition or Subtraction In addition or subtraction of the numerical values the final result should retain the least decimal place as in the various numerical values. e.g.,

If $l_1 = 4.326$ m and $l_2 = 1.50$ m

Then, $l_1 + l_2 = (4.326 + 1.50)$ m = 5.826 m

As l_2 has measured upto two decimal places, therefore

$l_1 + l_2 = 5.83$ m

(ii) In Multiplication or Division In multiplication or division of the numerical values, the final result should retain the least significant figures as the various numerical values. e.g., If length

$l = 12.5$ m and breadth $b = 4.125$ m.

Then, area $A = l \times b = 12.5 \times 4.125 = 51.5625$ m²

As l has only 3 significant figures, therefore

$A = 51.6$ m²

Rules of Rounding Off Significant Figures

1. If the digit to be dropped is less than 5, then the preceding digit is left unchanged. e.g., 1.54 is rounded off to 1.5.
2. If the digit to be dropped is greater than 5, then the preceding digit is raised by one. e.g., 2.49 is rounded off to 2.5.
3. If the digit to be dropped is 5 followed by digit other than zero, then the preceding digit is raised by one. e.g., 3.55 is rounded off to 3.6.
4. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd and left unchanged if it is even. e.g., 3.750 is rounded off to 3.8 and

4.650 is rounded off to 4.6.

Error

The lack in accuracy in the measurement due to the limit of accuracy of the instrument or due to any other cause is called an error.

1. Absolute Error

The difference between the true value and the measured value of a quantity is called absolute error.

If $a_1, a_2, a_3, \dots, a_n$ are the measured values of any quantity a in an experiment performed n times, then the arithmetic mean of these values is called the true value (a_m) of the quantity.

The absolute error in measured values is given by

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

.....

$$\Delta a_n = a_m - a_n$$

2. Mean Absolute Error

The arithmetic mean of the magnitude of absolute errors in all the measurement is called mean absolute error.

3. Relative Error The ratio of mean absolute error to the true value is called relative

4. Percentage Error The relative error expressed in percentage is called percentage error.

Propagation of Error

(i) Error in Addition or Subtraction Let $x = a + b$ or $x = a - b$

If the measured values of two quantities a and b are $(a \pm \Delta a)$ and $(b \pm \Delta b)$, then maximum absolute error in their addition or subtraction.

$$\Delta x = \pm(\Delta a + \Delta b)$$

(ii) Error in Multiplication or Division Let $x = a \times b$ or $x = (a/b)$.

If the measured values of a and b are $(a \pm \Delta a)$ and $(b \pm \Delta b)$, then maximum relative error

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

Physical world and Measurement

1. The distance covered by particle in time t , is given by $x = a + bt + ct^2 + dt^3$, then the dimensions of a, b, c and d are

- (a) $a=L, b=LT^{-1}, c=LT^{-2}, d=LT^{-3}$ (b) $a=L, b=LT^{-2}, c=LT^{-2}, d=LT^{-3}$
(c) $a=L, b=LT^{-2}, c=LT^{-1}, d=LT^{-2}$ (d) $a=L^2, b=LT^{-1}, c=LT^{-2}, d=LT^{-2}$

2. If $X = a + bt$, where X is in meter and t is in second. What is the unit of a and b ?

- (a) **m and m/sec²** (b) m/sec² and m (c) m/sec and m/sec (d) none of these.

3. In Van der Waal's equation $(P + a/V^2)(V - b) = RT$, Determine the dimensions of a and b .

- (a) $a = [ML^3 T^{-1}]$ and $b = [M^0 L^3 T^0]$. (b) $a = [ML^2 T^{-1}]$ and $b = [M^0 L^3 T^0]$ (c) **$a = [ML^5 T^{-2}]$ and $b = [M^0 L^3 T^0]$**
(d) $a = [ML^5 T^{-2}]$ and $b = [M^0 L^2 T^0]$

4. A Physical quantity P is given by $P = a^3 b^2 / \sqrt{cd}$, The percentage error of measurements in a, b, c and d are 1%, 3%, 4%, and 2% respectively, what is the percentage error in P

- (a) 10% (b) 15% (c) 12% (d) **13%**

5. To study the flow of a liquid through a narrow tube, the following formula is used: $\eta = \frac{\pi p r^4}{8 V l}$, where the letters have their usual meanings. The values of p, r, V and l are measured to be 76 cm of Hg, 0.28, $1.2 \text{ cm}^3 \text{ s}^{-1}$ and 18.2 cm respectively. If these quantities are measured to the accuracies of 0.5 cm Hg, 0.01 cm, $0.1 \text{ cm}^3 \text{ s}^{-1}$ and 0.1 cm respectively, find the percentage error in the value η .

- (a) 18.5% (b) **23.83%** (c) 12% (d) 22%

6. Which physical quantity that has no unit and no dimensions in the following:-

- (a) force (b) work (c) **strain** (d) gravitational constant

7. A physical quantity X is given by $X = A^2B^3/C\sqrt{D}$, If the percentage errors of measurement in A, B, C and D are 4%, 2%, 3% and 1% respectively, then calculate the % error in X

- (a) **17.5%** (b) 13.5% (c) 12% (d) 19%

8. Give the dimensional formula of thermal conductivity.

- (a) **$[k] = [MLT^{-1}K^{-1}]$** (b) $[k] = [M^{-1}LT^{-1}K^{-1}]$ (c) $[k] = [MLT^{-1}K^{-2}]$ (d) $[k] = [MLT^{-2}K^{-2}]$

9. C.V. Raman got Nobel his prize for experiment on

- (a) dispersion of light (b) reflection of light (c) deflection of light (d) **scattering of light**

10. Which of the following is true for the solid angle?

- (a) $\delta\omega = \frac{\delta A \cos\theta}{r^2}$ (b) $\delta\omega = \frac{\delta A \cos^2\theta}{r^2}$ (c) $\delta\omega = \frac{\delta A \cos\theta}{r^2}$ (d) $\delta\omega = \frac{\delta A \cos\theta}{r^3}$

11. Dimensions of Hubble's constant are

- (a) T^{-1} (b) MLT^{-4} (c) $M^0L^0T^{-2}$ (d) MLT^{-1}

12. The units of planks constant are

- (a) J/s (b) Js^2 (c) **Js** (d) Js^{-2}

13. Which of the following have same dimensions?

- (a) torque and force (b) **torque and potential energy**
(c) Potential energy and force (d) Planck's constant and momentum

14. Which one of the following is a dimensional constant?

- (a) relative density (b) **gravitational constant** (c) refractive index (d) Poisson ratio.

15. Which one the following five parameters have the same dimensions?

1. energy density 2. refractive index 3. dielectric constant 4. Young's modulus 5. magnetic field

- (a) 2 and 4 (b) 3 and 5 (c) **1 and 4** (d) 1 and 5

16. Which one of the dimensionless quantity

- (a) **strain** (b) stress (c) specific heat (d) Quantity of heat

17. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-5}

- (a) 4, 4, 2 (b) **5, 1, 2** (c) 5, 1, 5 (d) 5, 5, 2

18. The physical quantities not having the same dimensions are

- (a) torque and work (b) **momentum and Planck's constant** (c) stress and young's modulus
(d) speed and $\frac{1}{\sqrt{\mu\epsilon}}$ ($\mu = \text{absolute permiability}, \epsilon = \text{absolute permittivity}$)

19. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then the error in the value of wire is

- (a) **6%** (b) Zero (c) 1% (d) 3%

20. A cube has a side of length $1.2 \times 10^{-2}m$. Calculate its volume.

- (a) **$1.7 \times 10^{-6}m^3$** (b) $1.73 \times 10^{-6}m^3$ (c) $1.0 \times 10^{-6}m^3$ (d) $1.732 \times 10^{-6}m^3$

21. The dimensions of $\frac{1}{2}\epsilon^2 E$ (Where ϵ – permittivity of free space and E electric field

- (a) $[MLT^{-1}]$ (b) ML^2T^{-2} (c) **$ML^{-1}T^{-2}$** (d) ML^2T^{-1}

22. Dimensions of impulse are equal to the that

- (a) Pressure (b) **linear momentum** (c) force (d) angular momentum

23. The dimensional formula of magnetic flux

- (a) $[M LT^{-2} A^2]$ (b) $[M^0 L^{-2} T^{-1} A^2]$ (c) **$[M L^2 T^{-2} A^{-1}]$** (d) $[M L^2 T^{-1} A^2]$

24. The dimensional formula of magnetic permeability of free space μ_0
 (a) $[M LT^{-2} A^2]$ (b) $[M^0 L^1 T^1]$ (c) $[M^0 L^2 T^{-1} A^2]$ (d) none of these
25. Parsec is the unit of
 (a) time (b) **distance** (c) velocity (d) angular momentum
26. The difference in the length of a mean solar day and sidereal day is about
 (a) 1 minute (b) **4 minutes** (c) 15 minutes (d) 56 minutes
27. SONAR emits which of the following waves?
 (a) radio (b) light (c) **ultrasound** (d) none of these
28. If error in radius is 3% what is error in volume of sphere?
 (a) 3% (b) 27% (c) **9%** (d) 6%
29. The unit of a van der Waals gas equation is
 (a) $\text{atm L}^{-2} \text{mol}^2$ (b) **$\text{atm L}^2 \text{ per mol}$** (c) $\text{atm L}^{-1} \text{mol}^{-2}$ (d) $\text{atm L}^2 \text{mol}^{-2}$
30. Which of the following quantities can be written in $\text{kgm}^2 \text{A}^{-2} \text{s}^{-3}$?
 (a) **Resistance** (b) Inductance (c) Capacitance (d) Magnetic flux

Answer -1(a) 2.(a) 3.(c) 4.(d) 5.(b) 6.(c) 7.(a) 8.(a) 9.(d) 10.(a) 11.(c) 12.(c) 13.(b) 14.(b) 15.(c) 16.(a) 17.(b) 18.(b) 19.(a) 20.(a) 21.(c) 22.(b) 23.(c) 24.(a) 25.(b) 26(b) 27.(c) 28.(c) 29.(b) 30.(a)

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

- 1...If L,C,R represent inductance ,capacitance and resistance respectively, the combinations having dimensions of frequency are
 (a) $\frac{1}{\sqrt{LC}}$ (b) $\frac{L}{C}$ (c) $\frac{R}{L}$ (d) $\frac{R}{C}$
2. Which of the following combinations have the dimensions of time? LC,R represent inductance ,capacitance and resistance
 (a) RC (b) \sqrt{LC} (c) $\frac{R}{L}$ (d) C/L
3. In terms of potential difference V ,electric current I, absolute permittivity ϵ and permeability μ and speed of light c, the dimensionally correct equations is (are)
 (a) $\mu_0 I^2 = \epsilon_0 V^2$ (b) $\mu_0 V = \epsilon_0 I$ (c) **$I = \epsilon_0 c V$** (d) $\mu_0 c I = \epsilon_0 V$
4. The dimensional formula of the quantities in one(or more) of the following pairs of quantities are the same .Identify the pairs(s)
 (a) **Torque and work** (b) Angular momentum and work (c) Energy and Young's modulus
 (d) **Light year and wave length.**
5. Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation moves it by two divisions on the linear scale. Then
 (a) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of screw gauge is 0.01 mm.
 (b) **If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of screw gauge is 0.005 mm.**

(c) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01mm.

(d) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers the least count of the screw gauge is 0.005mm
6. The SI unit of inductance, the henry can be written as

(a) weber//ampere (b)volt-sec/amp (c)joule/ampere² (d)ohm-second

7. The pairs of physical quantities that have the same dimensions are

(a) Reynold's number and coefficient of friction (b) Curie and frequency of a light year (c) Latent heat an gravitational potential (d) Plank's constant an torque

ANSWER

1. (a) and (c) 2 .(a) and (b) 3.(a) and (c) 4. (a) and(d) 5. (b) and (c)6.All are correct 7.All are correct

ANSWERS CHALLENGING TYPE QUESTIONS WITH ANSWER.

Q1. What are the dimensions of $1/\mu_0\epsilon_0$, where symbols have their usual meaning.

Q2.What is the dimensions of $(1/2)\epsilon_0E^2$, Where E electric field and ϵ_0 permittivity of free space.

Q3. The pairs of physical quantities that have the same dimensions are:

(a) Reynolds's number and coefficient of friction,
(b) Curie and frequency of a light wave
(c) Latent heat and gravitational potential
(d) Planck's constant and torque.

Q4. If L, C, R represent inductance, capacitance and resistance respectively, the Combinations having dimensions of frequency are

(a) $1/\sqrt{CL}$ (b) L/C (c) R/L (d) R/C

Q5. If the error in radius is 3%, what is error in volume of sphere?

(a) 3 % (b) 27 % (c) 9 % (d) 6 %

ANSWERS

1. Ans : [M⁰ L² T⁻²]

2. Ans : [M¹ L⁻¹ T⁻²]

3. Ans : (a), (b).

4. Ans : (a) and (c).

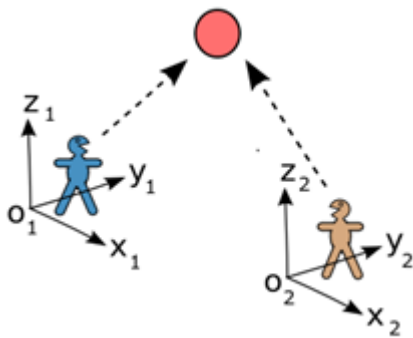
5. Ans : (c) 9%.

UNIT-II

KINEMATICS

Introduction to Motion in One Dimension

Frame of Reference:- A frame of reference is a set of coordinate axes which is fixed with respect to a space point (a body or an object can also be treated as a point mass therefore it can become a site for fixing a reference frame), which we have arbitrarily chosen as per our observer's requirement. The essential requirement for a frame of reference is that, it should be rigid.



Position Vector:- given by vector $\vec{r} = xi + yj + zk$.

In one-dimensional motion: vector $\vec{r} = xi$, $y = z = 0$ (along x-axis)

In two-dimensional motion: vector $\vec{r} = xi + yj$ (in x-y plane $z = 0$)

Displacement:- $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$

Velocity Vector in Non Uniform Motion:-

$$\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t}$$

The ratio of total distance traveled and time taken during the motion is called average speed. Average speed is a scalar quantity.

If at any time t_1 position vector of the particle is \vec{r}_1 and at time t_2 position vector is \vec{r}_2 then for

this interval
$$\vec{v}_{avg} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$$

Instantaneous velocity

Instantaneous velocity is defined as the rate of change of displacement.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} \Rightarrow \vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

Velocity

The velocity at any instant is obtained from the average velocity shrinking the time interval closer to zero. As Δt tends to zero, the average velocity approaches a limiting value, which is the velocity at that instant, called instantaneous velocity, which is a vector quantity, mathematically we can define it as

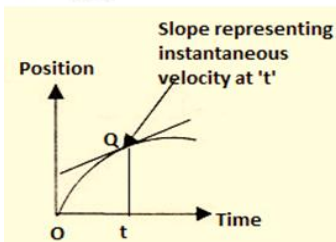
$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

The magnitude v of the instantaneous velocity is called the speed and is simple the absolute value

of \vec{v} i.e. $|\vec{v}| = \left| \frac{d\vec{x}}{dt} \right|$

In the example related with figure given below, the instantaneous velocity is

$$\vec{v} = \frac{d\vec{x}}{dt}$$



Hence instantaneous velocity is the rate at which a particle's position is changing with respect to time at a given instant. The velocity of a particle at any instant is the slope (tangent) of its position curve at the point representing that instant of time, as shown in figure above.

Acceleration

Acceleration is the rate of change of velocity with time. The concept of acceleration is understood in non-uniform motion. It is a vector quantity.

Average acceleration is the change in velocity per unit time over an interval of time.

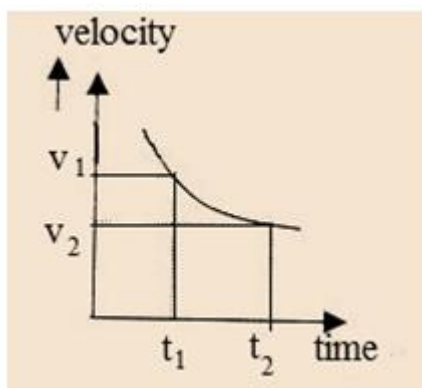
$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Instantaneous acceleration is defined as

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\Rightarrow \vec{a} \frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

Acceleration Vector in Non Uniform Motion



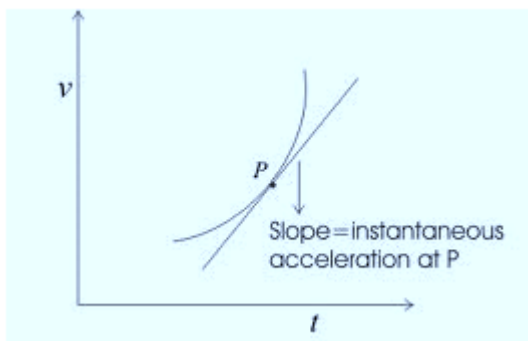
Suppose that at the instant t_1 a particle as in figure above, has velocity \vec{v}_1 and at t_2 , velocity is \vec{v}_2 . The average acceleration \vec{a}_{av} during the motion is defined as

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Variable Acceleration

The acceleration at any instant is obtained from the average acceleration by shrinking the time interval closer zero. As Δt tends to zero average acceleration approaching a limiting value, which is the acceleration at that instant called instantaneous acceleration which is vector quantity.

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$



i.e. the instantaneous acceleration is the derivative of velocity.

Hence instantaneous acceleration of a particle at any instant is the rate at which its velocity is changing at that instant. Instantaneous acceleration at any point is the slope of the curve $v(t)$ at that point as shown in figure above.

Equations of Motion

The relationship among different parameter like displacement velocity, acceleration can be derived using the concept of average acceleration and concept of average acceleration and instantaneous acceleration.

When acceleration is constant, a distinction between average acceleration and instantaneous acceleration loses its meaning, so we can write

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t - t_0} = \frac{d\vec{v}}{dt}$$

$$\vec{v} - \vec{v}_0 = \vec{a} t \quad \dots\dots\dots (2)$$

This is the first useful equation of motion.

Similarly for displacement

$$\vec{x} = \vec{x}_0 + \langle \vec{v} \rangle t \quad \dots\dots\dots (3)$$

in which \vec{x}_0 is the position of the particle at t_0 and v_{av} is the average velocity between t_0 and later time t . If at t_0 and t the velocity of particle is

$$\langle \vec{v} \rangle = \frac{1}{2} (\vec{v}_0 + \vec{v})$$

$$= \frac{1}{2} [\vec{v}_0 + \vec{v}_0 + \vec{a} t]$$

$$\langle \vec{v} \rangle = \vec{v}_0 + \vec{a} t/2 \quad \dots\dots\dots (4)$$

From equation (3) and (4), we get

$$\vec{x} - \vec{x}_0 = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \quad \dots\dots\dots (5)$$

This is the second important equation of motion.

Now from equation (2), square both side of this equation we get

$$v^2 = v_0^2 + a^2 t^2 + 2v_0 a t = v_0^2 + 2 a t + [v_0 + a t/2]$$

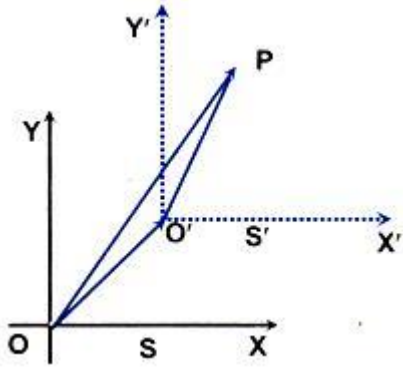
$$= v_0^2 + 2 a t < v > \text{ (Use equation 4)}$$

Use equation (3), to get

$$\vec{v}^2 = \vec{v}_0^2 + 2a(\vec{x} - \vec{x}_0) \dots\dots\dots (6)$$

This is another important equation of motion.

Relative Velocity:- Relative velocity of a body A with respect to another body B, when both are in motion, is the velocity with which A appears to move to B.

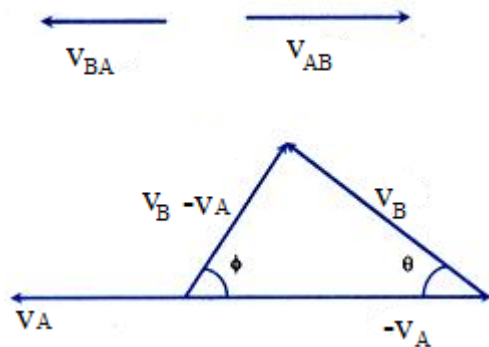


$$\vec{O'P} = \vec{OP} - \vec{OS}$$

$$\Rightarrow \vec{r}_{p,s'} = \vec{r}_{p,s} - \vec{r}_{s',s} \quad \Rightarrow \quad \frac{d}{dt}(\vec{r}_{p,s'}) = \frac{d}{dt}(\vec{r}_{p,s}) - \frac{d}{dt}(\vec{r}_{s',s})$$

$$\Rightarrow \vec{v}_{p,s'} = \vec{v}_{p,s} - \vec{v}_{s',s} \quad \Rightarrow \quad \vec{v}_{p,s'} = \vec{v}_{p(\text{absolute})} - \vec{v}_{s'(\text{absolute})}$$

Physical Significance of Relative Velocity



Therefore,

$$\vec{v}_{AB} = -\vec{v}_{BA}$$

In general, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

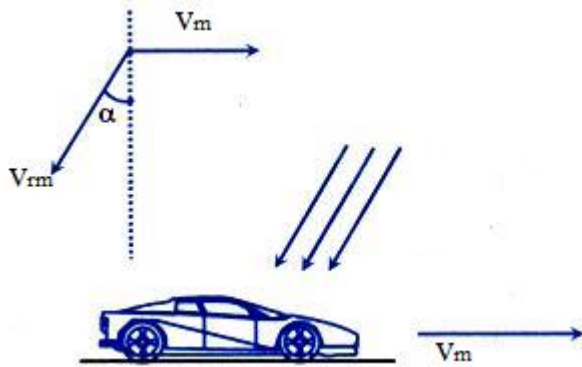
So,

$$|\vec{v}_{BA}| = |\vec{v}_{AB}|$$

$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

$$\text{and } \theta = \tan^{-1}\{(v_B \sin\theta)/(v_A - v_B \cos\theta)\}$$

Relative Motion between Rain and Man



We know that, $v_r = v_{r0}$ = velocity of rain w.r.t. ground, $v_m \equiv v_{mg}$.

Velocity of man w.r.t. ground and

$$\vec{v}_{rm} = \vec{v}_{rm} - \vec{v}_m$$

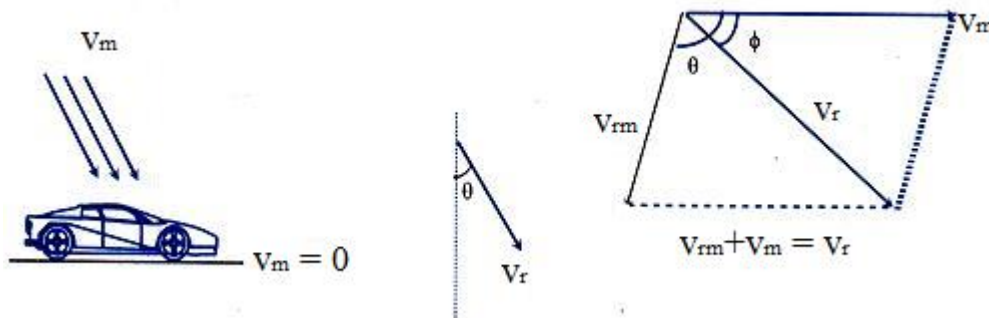
velocity of rain w.r.t. man.

$$\text{So, } \vec{v}_r = \vec{v}_{rm} + \vec{v}_m$$

That means the vector addition of the velocity of rain with respect of man (\vec{v}_{rm}) and the velocity of man (vehicle) (\vec{v}_m) yield the actual velocity of rain \vec{v}_r . The magnitude and direction of \vec{v}_r can be given as,

$$v_r = \sqrt{(v_{rm})^2 + (v_m)^2 + 2v_{rm} v_m \cos\theta}$$

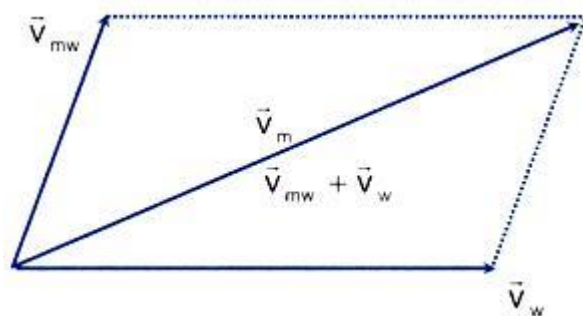
$$? = \tan^{-1}((v_{rm} \sin\theta)/(v_{rm} \cos\theta + v_m)) \text{ with horizontal } \vec{v}_m$$



Relative Motion of a Swimmer in Flowing Water

Take \vec{v}_m = velocity of man

\vec{v}_w = velocity of flow of river,



\vec{v}_{mw} = velocity of swimmer w.r.t. river

\vec{v}_m can be found by the velocity addition of \vec{v}_{mw} and \vec{v}_w .

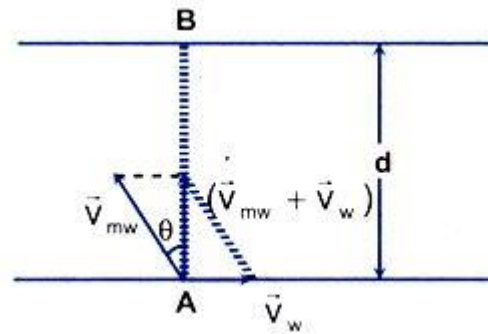
Since $\vec{v}_{mw} = \vec{v}_m - \vec{v}_w$.

$$\vec{v}_m = \vec{v}_{mw} + \vec{v}_w$$

Crossing of the River with Minimum Drift

Case 1: $v_{mw} > w$

A man intends to reach the opposite bank at the point directly opposite to the stationary point. He has to swim at angle θ with a given speed \vec{v}_{mw} w.r.t. water, such that his actual velocity \vec{v}_m will direct along AB, that is perpendicular to the bank (or velocity of water \vec{v}_w).



=> For minimum drift, $\vec{v}_m \perp \vec{v}_w$

You can realize the situation by a simple example. If you want to reach the directly opposite point or cross the river perpendicularly, a man, that is to say, Hari, must report you that, you are moving perpendicular to the shore. What does this report signify? Since Hari observes your actual velocity (\vec{v}_m) to be perpendicular to the bank \vec{v}_m is perpendicular to \vec{v}_w .

Observing the vector-triangle $v_w = v_{mw} \sin\theta$ & $v_m = v_{mw} \cos\theta$

$$\Rightarrow \theta = \sin^{-1}(v_w/v_{mw}) \quad \& \quad v_m = \sqrt{(v_{mw})^2 - (v_w)^2}$$

=> The time of crossing, $t = d/v_m$

$$\Rightarrow t = d/\sqrt{(v_{mw})^2 - (v_w)^2}$$

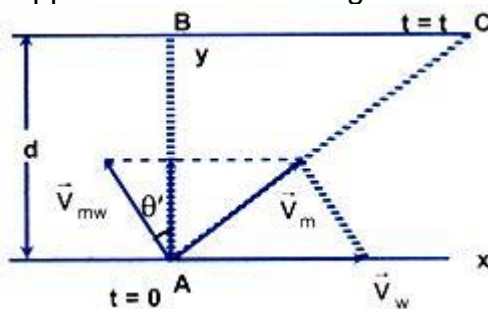
Case 2 : $v_w > v_{mw}$

Let the man swim at an angle θ' with normal to the bank for minimum drift. Suppose the drift is equal to zero. For zero drift, the velocity of the man along the bank must be zero.

$$\Rightarrow v_m = v_w - v_{mw} \sin\theta' = 0$$

This gives, $\sin\theta' = v_w / v_{mw}$, since $v_w > v_{mw}$, $\sin\theta' > 1$ which is impossible. Therefore, the drift cannot be zero.

Now, let the man swim at an angle θ with the normal to the bank to experience minimum drift. Suppose that the drifting of the man during time t when he reaches the opposite bank is,



$$BC = x$$

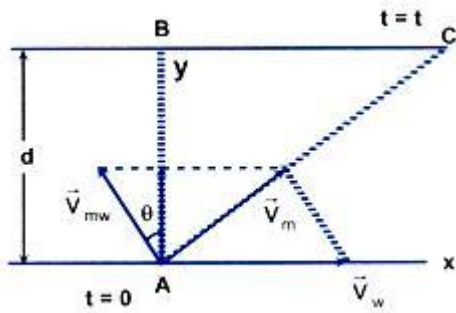
$$x = (v_m)t \quad \dots (1)$$

$$\text{where } t = AB/((v_m) \cos\theta) = d/(v_{mw} \cos\theta) \quad \dots (2)$$

$$\text{and } (v_m)t = v_w - v_{mw} \sin\theta \quad \dots (3)$$

Using (1), (2) & (3), we obtain

$$x = (v_w - v_{mw} \sin\theta) d/(v_{mw} \cos\theta)$$



$$x = (v_w/v_{mw} \sec \theta - \tan \theta)d \quad \dots (4)$$

For x to be minimum,

$$dx/d\theta = (v_w/v_{mw} \sec \theta - \tan \theta - \sec^2 \theta)d = 0$$

$$v_w/v_{mw} \tan \theta = (\sec \theta) \Rightarrow \sin \theta = v_{mw}/v_w$$

$$\theta = \sin^{-1}(v_{mw}/v_w)$$

Substituting the value of θ in (4), we obtain,

$$x = [\sqrt{(v_w^2 - v_{mw}^2)}/v_{mw}] d$$

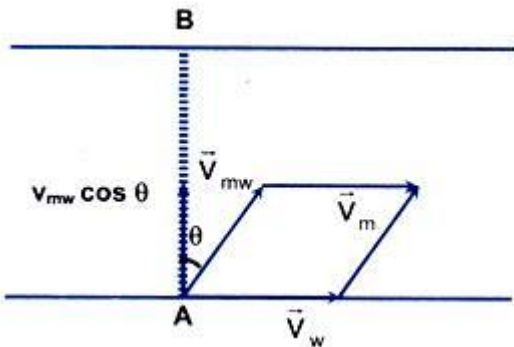
Crossing of the River in Minimum Time

Case 1: To reach the opposite bank for a given v_{mw}

Let the man swim at an angle θ with AB. We know that the component of the velocity of man along shore is not responsible for its crossing the river. Only the component of velocity of man (v_m) along AB is responsible for its crossing along AB.

The time of crossing = $t = AB/(v_{mw} \cos \theta)$

Time is minimum when $\cos \theta$ is maximum.



The maximum value of $\cos \theta$ is 1 for $\theta = 0$.

That means the man should swim perpendicular to the shore.

$$\Rightarrow \vec{v}_{mw} \perp \vec{v}_w$$

$$\Rightarrow \text{Then } t_{\min} = d/(v_{mw} \cos \theta)|_{(\theta=0)} = d/v_{mw} \Rightarrow t_{\min} = d/v_{mw}$$

Case 2:

To reach directly opposite point on the other bank for a given v_{mw} & velocity v of walking along the shore.

To attain the direct opposite point B in the minimum time, let the man swim at an angle θ with the direction AB. The total time of journey t = the time taken from A to C and the time taken from C to B.

$$\Rightarrow t = t_{AC} + t_{CB}$$

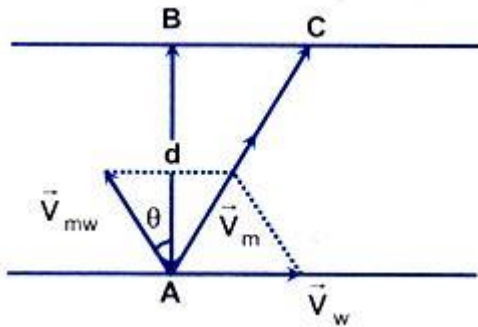
where $t_{AC} = AB/v_{mw} \cos \theta$ & $t_{CB} = BC/v$ where v = walking speed of the man from C to B.

$$\Rightarrow t = AB/v_{mw} \cos \theta + BC/v$$

Again $BC = (v_m)t$

$$\Rightarrow BC = (v_w - v_{mw}\sin\theta) (AB/v_{mw}\cos\theta)$$

Using (1) & (2) we obtain,



$$t = AB/v_{mw}\cos\theta + ((v_w - v_{mw}\sin\theta)/v(v_{mw}\cos\theta))$$

$$\Rightarrow t = AB[(1+v_w/v)\sec\theta/v_{mv} - \tan\theta/v]$$

$$\Rightarrow t = d/v_{mv}[(1+v_w/v)\sec\theta/v_{mv} - \tan\theta/v]$$

Putting $dt/d\theta = 0$, For minimum t we get,

$$dt/d\theta = d/d\theta[d/v_{mv} (1+v_w/v) \sec\theta/v_{mv} - \tan\theta/v]$$

$$= [\sec\theta/v_{mv} - \tan\theta/v (1+v_w/v) (\sec^2\theta)/v] = 0$$

$$\Rightarrow \tan\theta/v_{mv} (1+v_w/v) \sec\theta/v$$

$$\Rightarrow \sin\theta = (v_{mw}/v+v_w)$$

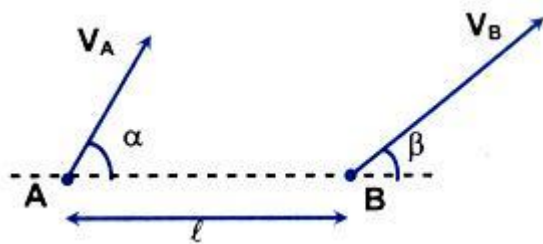
$$\Rightarrow \theta = \sin^{-1}(v_{mw}/v+v_w)$$

This expression is obviously true when $v_{mw} < v + v_w$.

Velocity of Separation/Approach

Let there be two particles A and B with velocity \vec{v}_A and \vec{v}_B at any instant as visualized from ground frame.

If we visualize the motion of B from frame of A the velocity of particle B would be $\vec{v}_B - \vec{v}_A$.



If α, β be the angle made with line AB,

then, $V_B \cos \beta - V_A \cos \alpha$ is relatively velocity of B w.r.t. A along line AB.

- If $V_B \cos \beta - V_A \cos \alpha > 0$; it is called as velocity of separation.
- If $V_B \cos \beta - V_A \cos \alpha < 0$; it is called as velocity of approach.

$V_B \sin \beta - V_A \sin \alpha$ is relative velocity of B w.r.t. A along direction perpendicular to AB. If length of AB is,

then, angular velocity B w.r.t. A is $(V_B \sin \beta - V_A \sin \alpha)/l$

and relative angular velocity = $(V_B \sin \beta - V_A \sin \alpha)/l$.

Motion in Two Dimensions

$$\vec{r} = x \hat{i} + y \hat{j}, \quad \mathbf{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} = v_x \hat{i} + v_y \hat{j}$$

$$\text{and } \vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} = a_x \hat{i} + a_y \hat{j}$$

Circular Motion:- The motion of a body is said to be circular if it moves in such a way that its distance from a certain fixed point always remains the same.

The rate of change of angular position is called angular velocity, ω , measured in radian per second.

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t}$$

$$= d\theta/dt = ds/rdt = v/r$$

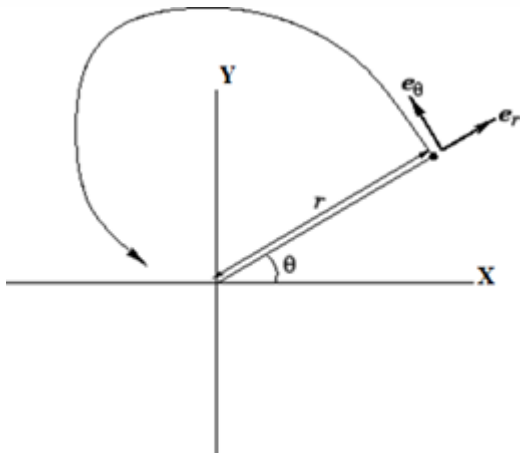
The rate of change of angular velocity is called angular acceleration, measured in rad/s^2 . Thus, the angular acceleration is

$$\alpha = d\omega/dt = d^2\theta/dt^2$$

$$a = \frac{v^2}{r}$$

Simulation for Car and Curves This animation is used to explain why a passenger slides to the "outside" of a curve while riding inside a car is NOT an example of centrifugal forces. Instead it is a combination of centripetal force and inertia. It emphasizes that when an object moves to the outside of a circle it is because of a lack of enough centripetal force and inertia keeps it moving in a straight line.

Non uniform circular motion:-



Let us use the vector method to discuss non-uniform circular motion.

In the side figure, \hat{e}_r and \hat{e}_θ are unit vectors along radius and tangent vector respectively. In terms of e_r and e_θ the motion of a particle moving counter clockwise in a circle about the origin in figure 2.30 can be described by the vector equation.

$$\vec{v} = e_\theta v$$

In this case, not only \hat{e}_θ but v also varies with time. We can obtain instantaneous acceleration as,

$$\vec{a} = \frac{d\vec{v}}{dt} = \hat{e}_\theta \frac{d\theta}{dt} + v \left(\frac{d\hat{e}_\theta}{dt} \right)$$

$$\vec{a} = (d\vec{v})/dt = \hat{e}_\theta \theta d\theta/dt + v(d\hat{e}_\theta)/dt$$

Again,

$$\frac{d\hat{e}_\theta}{dt} = \frac{\hat{e}_r v^2}{r}$$

$$\vec{a} = \hat{e}_\theta a_T - \hat{e}_r \frac{v^2}{r} = a_T \hat{e}_\theta - a_R \hat{e}_r$$

Here, $a_T = dv/dt$ and $a_R = v^2/r$

The first term, $\hat{e}_\theta a_T$ is the vector component of \vec{A} that is tangential to the path of the particle and arises from a change in the magnitude of the velocity in circular motion, called tangential acceleration whereas a_R centripetal acceleration.

The magnitude of \vec{A} is

$$|\vec{a}| = \sqrt{(a_T^2 + a_R^2)}$$

Motion in a Straight Line with Acceleration

Non-Uniform Acceleration

The acceleration of a body is said to be non-uniform if its velocity changes by unequal amounts in equal intervals of time.

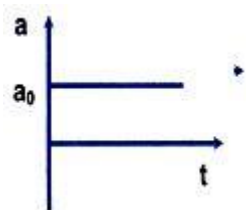
Average velocity

$$lt; v_{avg}gt; = \frac{\int_0^t v dt}{\int_0^t dt}$$

Average acceleration

$$lt; a_{avg}gt; = \frac{\int_0^t a dt}{\int_0^t dt}$$

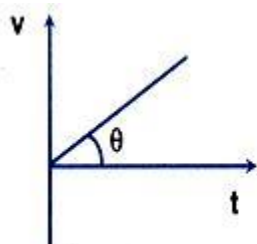
Analysis of Uniformly Accelerated Motion



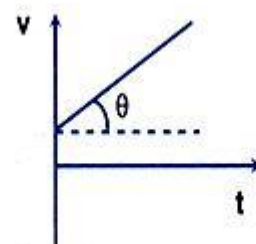
Case-I:

For uniformly accelerated motion with initial velocity u and initial position x_0 .

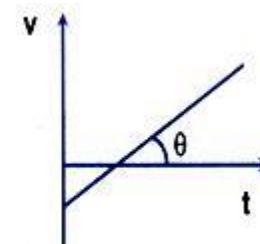
Velocity Time Graph



If $u = 0$



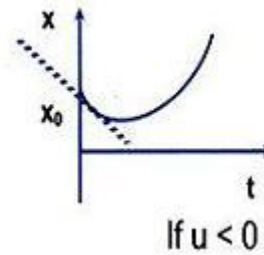
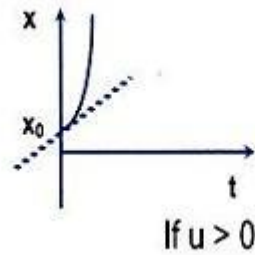
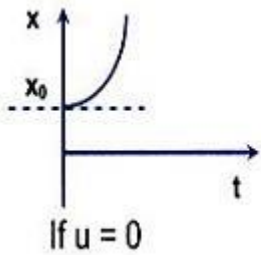
If $u > 0$



If $u < 0$

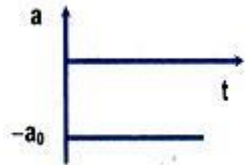
In every case $\tan\theta = a_0$

Position Time Graph



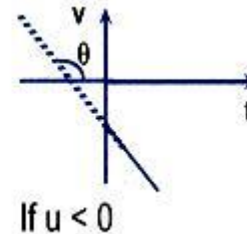
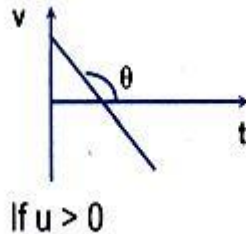
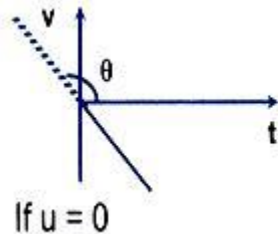
Initial position x of the body in every case is $x_0 (> 0)$

Case II:



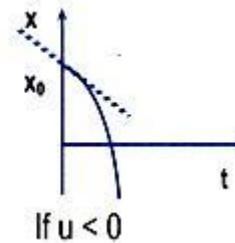
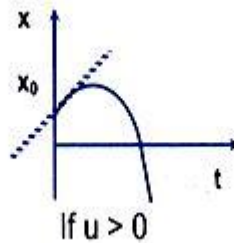
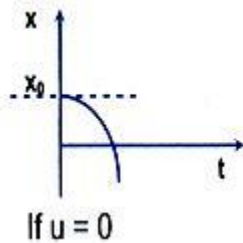
For uniformly retarded motion with initial velocity u and initial position x_0 .

Velocity Time Graph



In every case $\tan\theta = -a_0$

Position Time Graph



Initial position x of the body in every case is $x_0 (> 0)$

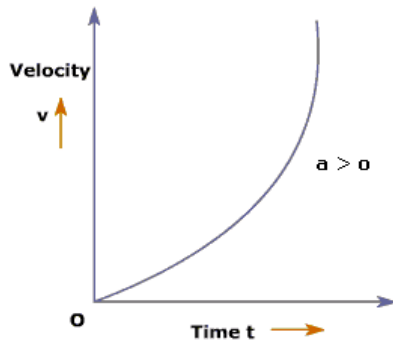
Acceleration

$$a_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Instantaneous acceleration

$$\Rightarrow \vec{a} \frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

Acceleration Vector in Non Uniform Motion



Equations of Motion

$$\vec{v} - \vec{v}_0 = \vec{a}t$$

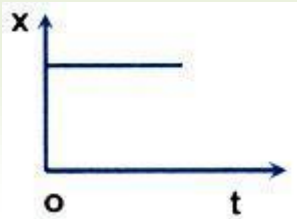
$$\vec{x} - \vec{x}_0 = \vec{v}_0t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

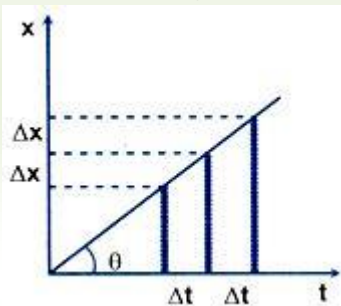
Graphical Representation and Equations of Motion

Position - Time Graph

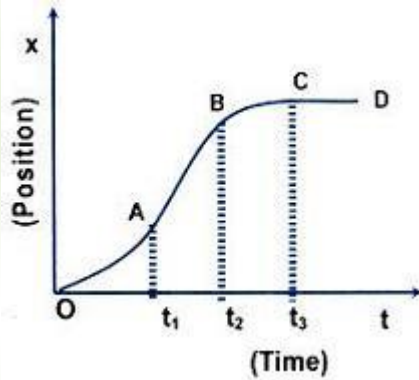
- (i) In this case, position (x) remains constant but time changes



- (ii) When the x - t graph is a straight line inclined at some angle ($\theta \neq 0$) with the time axis,

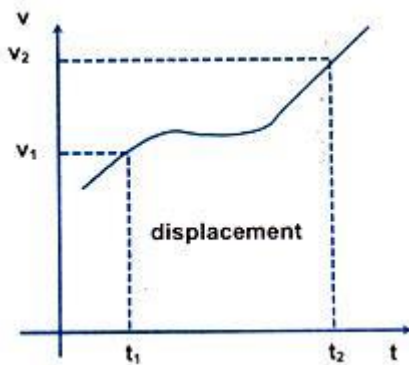


- (iii) When the x - t graph is a curve, motion is not uniform



The Velocity - Time Graph

The velocity-time graph gives three types of information.



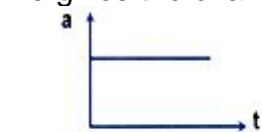
- (i) The instantaneous velocity.
- (ii) The slope of the tangent to the curve at any point gives instantaneous acceleration.
 $a = dv/dt = \tan \theta$
- (iii) The area under the curve gives total displacement of the particle.

$$s = \int_{t_1}^{t_2} v dt$$

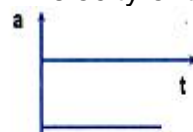
Now, let us consider the uniform acceleration. The velocity-time graph will be a straight line.

The acceleration-time graph:-

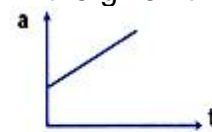
Acceleration time curves give information about the variation of acceleration with time. Area under the acceleration time curve gives the change in velocity of the particle in the given time interval.



Motion under uniform acceleration



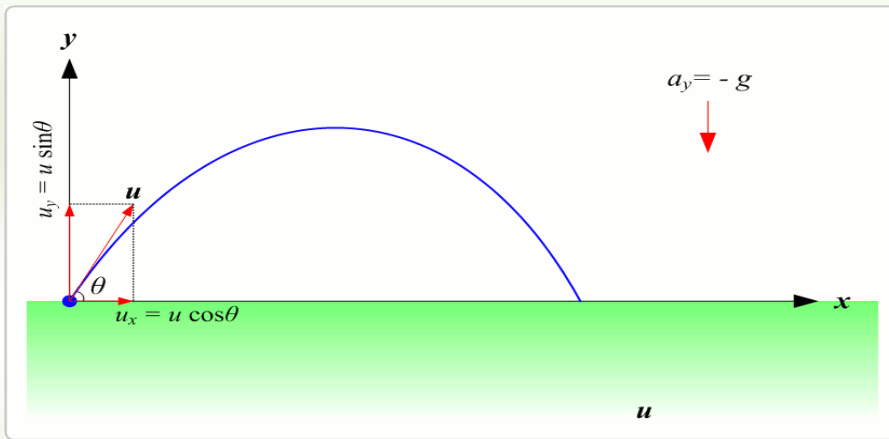
Motion under uniform retardation



Motion under variable acceleration

Motion of Projectile

A body projected into the space and is no longer being propelled by fuel is called a projectile



Velocity along x-axis = $u_x = u \cos \theta$

Acceleration along x-axis $a_x = 0$

Velocity along y-axis = $u_y = u \sin \theta$

Acceleration along y-axis $a_y = -g$

Here we use different equation of motions of one dimension derived earlier to get the different parameters.

$$\vec{v} = \vec{v}_0 - \vec{g}t \quad \dots\dots (a)$$

$$\vec{y} - \vec{y}_0 = \vec{v}_0 t - \frac{1}{2} \vec{g} t^2 \quad \dots\dots (b)$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

Total Time of Flight

$$t = \frac{2u \sin \theta}{g}$$

Horizontal Range

$$X = \frac{u^2 \sin 2\theta}{g}$$

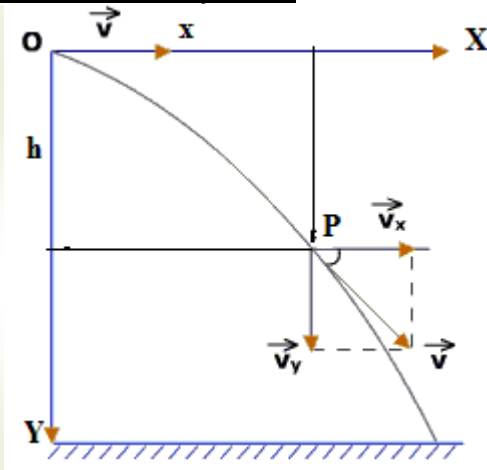
Maximum Height

$$H_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

Equation of Trajectory

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

Horizontal Projection:-



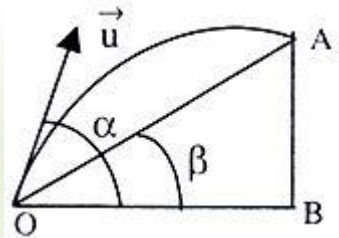
, $x = \text{horizontal distance covered in time } t = ut$ (1)

$y = \text{vertical distance covered in time } t = \frac{1}{2}gt^2$ (2)

Eliminate t from equations (1) and (2) then we get,

$$y = \frac{1}{2}(g/u^2) (x^2)$$

Projectile Motion on an inclined plane:-



(i) $u \cos (\alpha - \beta)$ along the plane

(ii) $u \sin (\alpha - \beta)$ perpendicular to the plane.

The acceleration due to gravity g can be resolved into two components:

(i) $g \sin \beta$ parallel to the plane

(ii) $g \cos \beta$ perpendicular to the plane.

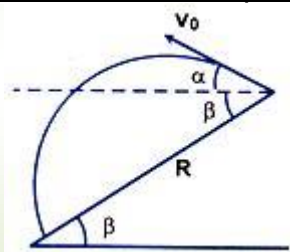
Time of Flight:-

$$\Rightarrow t = 2u \sin(\alpha - \beta) / g \cos \beta$$

Range:-

$$, OA = OB / \cos \beta = 2u^2 \sin(\alpha - \beta) \cos \alpha / g \cos \beta$$

Motion down the plane:-



Let the particle be thrown at a velocity v_0 at angle ' α ' with the horizontal as shown in figure.

$$v_0 \sin(\alpha+\beta)T - \frac{1}{2} g \cos\beta T^2 = 0 \text{ [for } y'=0]$$

$$\Rightarrow T = \frac{2v_0 \sin(\alpha+\beta)}{g \cos\beta}$$

$$R = v_0 \cos(\alpha+\beta)T + \frac{1}{2} g \sin\beta T^2 = \frac{v_0^2}{g} \left[\frac{\sin(2\alpha+\beta) + \sin\beta}{1 - \sin 2\beta} \right]$$

Since α is the variable and maximum value of sin function is 1, therefore for R to be maximum, $\sin(2\alpha+\beta) = 1$

$$\text{and } R_{\max} = \frac{v_0^2}{g} \left[\frac{1 + \sin\beta}{1 - \sin 2\beta} \right] = \frac{v_0^2}{g(1 - \sin\beta)} \text{ down the plane}$$

Motion Under Gravity

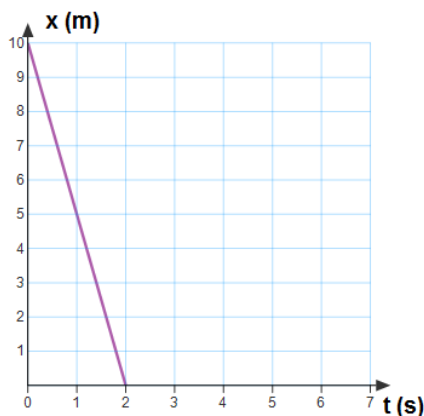
When the y-axis is chosen positive along vertically downward direction, we take \vec{g} as positive and use the equation as $v = u + gt$, $v^2 = u^2 + 2gh$, and $h = ut + \frac{1}{2}gt^2$

y-axis positive in the vertically upward direction and the set of equations reduces to

$$v = u - gt, \quad v^2 = u^2 - 2gh, \quad \text{and } h = ut - \frac{1}{2}gt^2$$

MCQ with one correct Answer

The following graph represents the position as a function of time of a moving object. Use this graph for questions 1 and 2.

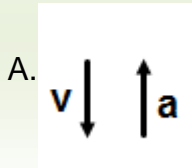


- Q.1 What is the initial position of the object?
 A. 2 m B. 4 m C. 6 m D. 8 m E. 10 m
- Q.2 What is the velocity of the object?
 A. 5 m/s B. -5 m/s C. 10 m/s D. -10 m/s E. 0 m/s
- Q.3 Four particles A, B, C and D are in motion. The velocities of one with respect to other are given as \vec{v}_{DC} is 20 m/s towards north, \vec{v}_{BC} is 20 m/s towards east and \vec{v}_{BA} is 20 m/s towards south. Then \vec{v}_{DA} is
- (a) 20 m/s towards north
 - (b) 20 m/s towards south
 - (c) 20 m/s towards east
 - (d) 20 m/s towards west

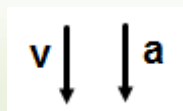
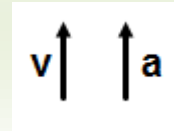
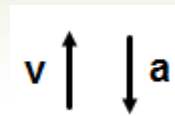
A ball is thrown straight up from point A, reaches a maximum height point B, and then falls back to point C, as illustrated by the picture to the right. Use this for questions 4 and 5.



Q.4 Which of the following is true about the direction the ball's velocity and acceleration between A and B?



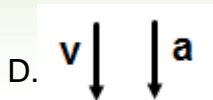
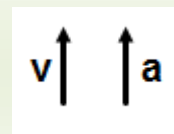
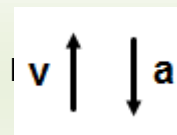
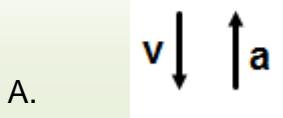
B.



E.

$v = 0 \quad a = 0$

Q.5 Which of the following is true about the direction the ball's velocity and acceleration between B and C?



$v = 0 \quad a = 0$

Q.6 A toy car is moving in a circular track covering equal distances in equal intervals of time. This shows that the toy car is moving with:

- (a) uniform velocity (b) varying speed
 (c) uniformly acceleration (d) uniform speed and acceleration of fixed magnitude

Q.7 A body is dropped from a height of 490 m from the ground. It will hit the ground after:

- (a) 10 s (b) 20 s (c) 30 s (d) 33 s

Q.8 When a body is moving in circular motion in a circular orbit at constant speed, it is in

- (a) equilibrium (b) not in equilibrium
 (c) unstable equilibrium (d) none of the above

- Q.9 When a body moves with a constant speed along a circle:
 (a) its velocity remains constant (b) no force acts on it
 (c) no work is done on it (d) no acceleration is produced
- Q.10 Two trains take 3 sec to pass one another when going in the opposite direction but only 2.5 sec if the speed of the one is increased by 50%. The time one would take to pass the other when going in the same direction at their original speed is
 (a) 10 sec (b) 12 sec (c) 15 sec (d) 18 sec
- Q.11 If a body of mass 3 kg is dropped downwards, after 1 sec another ball is dropped downwards from the same point. What is the distance between them after 3 sec.
 (a) 25 m (b) 20 m (c) 50 m (d) 9.8 m
- Q.12 Two bodies of different masses m_a and m_b are dropped from two different heights a and b . The ratio of the times taken by the two to drop through these distances is:
 (a) $a:b$ (b) $b:a$ (c) $a^2:b^2$ (d) $\sqrt{a}:\sqrt{b}$
- Q.13 The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is:
 (a) $2bV^3$ (b) $-2abV^2$ (c) $2aV^2$ (d) $-2aV^3$
- Q.14 A car has to cover the distance 60 km, if half of the total time it travels with velocity 80 km/h and in rest half time its speed becomes 40 km/h, the average speed of car will be,
 (a) 60 km/h (b) 80 km/h (c) 120 km/h (d) 180 km/h
- Q.15 In a straight line motion the distance travelled is proportional to the square root of the time taken. The acceleration of the particle is proportional to:
 (a) velocity (b) V^2 (c) V^3 (d) \sqrt{V}
- Q.16 A particle is thrown vertically upward. At its highest point, it has:
 (a) an upward velocity (b) downward velocity
 (c) an upward acceleration (d) a downward acceleration
- Q.17 A body is projected horizontally from the top of a tower 19.6 meter high. It reaches the ground in:
 (a) 1 sec (b) 2 sec (c) 2.5 sec (d) 5 sec
- Q.18 A stone is released with zero velocity from the top of a tower reaches the ground in 4 seconds, the height of the tower is about
 (a) 20 m (b) 40 m (c) 80 m (d) 16 m
- Q.19 A ball takes t second to fall from a height h_1 and $2t$ seconds to fall from a height h_2 . Then h_1/h_2 is:
 (a) 0.5 (b) 0.25 (c) 2 (d) 4
- Q.20 A particle moves in a straight line according to the relation
 $x = t^3 - 4t^2 + 3t$
 Find the acceleration of the particle at displacement equal to zero.
 (a) (-8,-2,10) (b) (-1,-2,10) (c) (8,2,10) (d) (1,2,10)
- Q.21 The radius vector of point x relative to origin varies with time as
 $r = a \cos(kt)i + b \sin(kt)j$
 Where a and b are constants and i and j are vectors along x and y axis. Which one of the following is the mean velocity vector?

- (a) $[a \cos(kt)\mathbf{i} - b \sin(kt)\mathbf{j}]/t$ (b) $[a \cos(kt)\mathbf{i} + b \sin(kt)\mathbf{j}]$
 (c) $[a \cos(kt)\mathbf{i} - b \sin(kt)\mathbf{j}]$ (d) $[a \cos(kt)\mathbf{i} + b \sin(kt)\mathbf{j}]/t$

Q.22 A body is freely falling under the action of gravity. It covers half the total distance in the last second of its fall. If it falls for n second, then value of n is

- (a) 2 (b) $2+\sqrt{2}$ (c) 3 (d) $2-\sqrt{2}$

Q.23 Two projectile A and B are having trajectory equation $y=a_1x-b_1x^2$ and $y=a_2x-b_2x^2$

If the range is same for both the projectile A and B then which of the following option is true

a. $\frac{a_1}{b_1} = \frac{a_2}{b_2}$

b. $\frac{a_1}{b_2} = \frac{a_2}{b_1}$

c. $a_1a_2=b_1b_2$

d. None of these

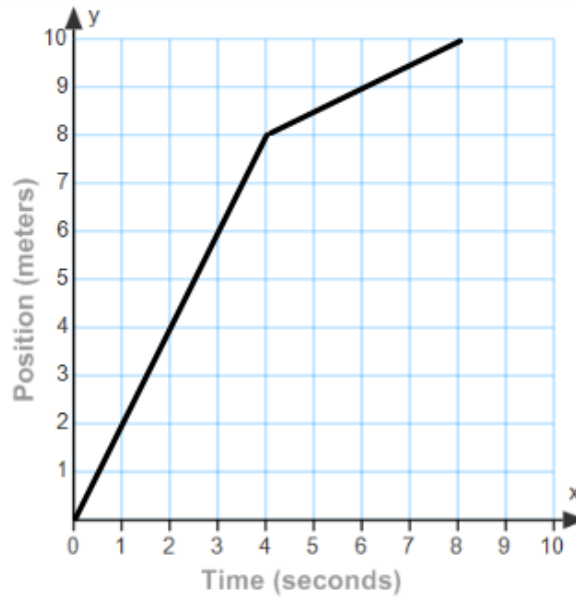
Q.24 When the projectile is at the highest point of its trajectory, the direction of its velocity and acceleration are

- a. parallel to each other
 b. anti parallel to each other
 c. Inclined to each other at 45
 d. Perpendicular to each other

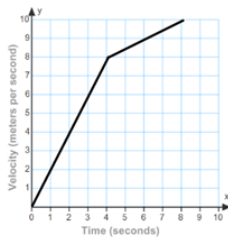
Q.25 A projectile has a range R and time of flight T . If the range is tripled by the increasing the speed of the projection, without changing the angle of projection then the time of the flight will become

- a. $T/\sqrt{3}$
 b. $T\sqrt{3}$
 c. $T/3$
 d. $3T$

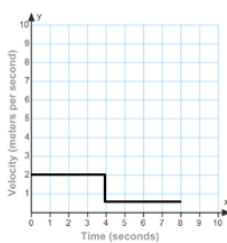
The position vs. time graph of a moving object is shown to the right. Use this graph to answer questions 26 through 29.



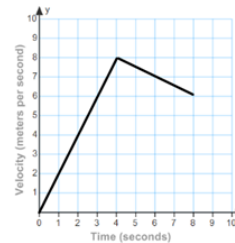
- Q.26 What is the average speed from 4 s to 8 s?
 A. 0.5 m/s B. 1 m/s C. 2 m/s D. 3 m/s E. 4 m/s
- Q.27 What is the object's position at 6 s?
 A. 2 m B. 1 m C. 3 m
 D. 7 m E. 9 m
- Q.28 What is the average acceleration from 4 s to 8 s?
 A. 0 m/s² B. 1 m/s² C. 2 m/s² D. 3 m/s² E. 4 m/s²
- Q.29 Which of the following is the velocity vs. time graph?



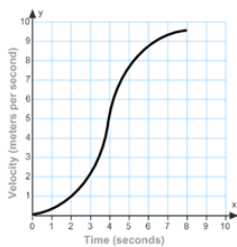
A.



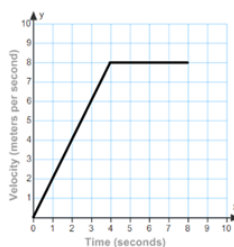
B.



C.



D.



E.

Q.30

Two cars A and B run at constant speed u_1 and u_2 along the highways intersecting at an angle θ . They start at $t=0$ at the intersection point. Find the time required to have distance s between the two cars

a) $\frac{s}{\sqrt{u_1^2 + u_2^2}}$

b) $\frac{s}{\sqrt{u_1^2 + u_2^2 - 2u_1u_2 \cos \theta}}$

c) $\frac{s}{\sqrt{u_1^2 + u_2^2 + 2u_1u_2 \cos \theta}}$

d) $\frac{s}{\sqrt{u_1^2 - u_2^2}}$

MCQ WITH MORE THAN ONE OPTION CORRECT

Q.1 This question contains statement-1 (Assertion) and Statement-2 (Reason).

Statement-1

A bus moving due north take a turn and starts moving towards east with same speed. There will be no change in the velocity of the bus.

Statement-2

Velocity is a vector quantity.

(A) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

Q.2 The velocity, displacement, acceleration of a particle in one dimensional motion is given as v_1, x_1, a_1 at $t=t_0$, v_2, x_2, a_2 at $t=t_0 + \Delta t$ which of the following is correct

$$a) x_2 = x_1 + v_1 \Delta t + \frac{1}{2} a_1 \Delta t^2$$

$$b) x_2 - x_1 = v_{avg} \Delta t$$

$$c) v_2 - v_1 = a_{avg} \Delta t$$

$$d) x_2 = x_1 + v_{avg} \Delta t + \frac{1}{2} a_{avg} \Delta t^2$$

Q.3 The displacement time equation for a particle in linear motion is given as

$$x = \frac{a}{b} (1 - e^{-bt})$$

which of the following option is correct

- a) The velocity and acceleration of the particle at $t=0$ is a and $-ab$ respectively
- b) The velocity will be decreasing as the time increases
- c) The displacement of the particle will fall between

$$0 \leq x \leq \frac{a}{b}$$

- d) The maximum acceleration in the motion is $-ab$

Q.4 Which one is wrong for a body having uniform circular motion?

- a). Speed of the body is constant
- b). Acceleration is directed towards the centre
- c). Velocity and Acceleration vector are having an angle 45°
- d). none of the above

Q.5 A body moves along a semicircular track of Radius R . Which of the following statement is true

- a. Displacement of the body is $2R$
- b. Distance travelled by the body is πR
- c. Displacement of the body is πR
- d. none of the above

Q.6 A particle is going moving along x -axis. Which of the following statement is false

- a. At time t_1 $(dx/dt)_{t=t_1}=0$, then $(d^2x/dt^2)_{t=t_1}=0$
- b. At time t_1 $(dx/dt)_{t=t_1} < 0$ then the particle is directed towards origin
- c. If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.
- d. At time t_1 $(d^2x/dt^2)_{t=t_1} < 0$ then the particle is directed towards origin

Q.7 A particle starts at time $t=0$ from $x=0$ along the positive x -axis with constant speed v . After time t , it return back towards the origin with the speed $2v$ and reaches the origin in $t/2$ sec. Which of the following is true for the whole process

- a. Average velocity is zero for the whole process
- b. Average speed is $4/3v$ for the whole process
- c. Displacement at time t is equal to vt

d. Displacement at time $3t/2$ is $2vt$

Q.8 The range of the projectile depends upon

- a. Angle of the projection
- b. Acceleration due to gravity
- c. Mass of the projectile
- d. magnitude of the velocity of projection

Q.9 Two bullets A and B are fired horizontally with speed v and $2v$ respectively. Which of the following is true

- a. Both will reach the ground in same time
- b. Bullet with speed $2v$ will cover more horizontal distance on the ground
- c. B will reach the ground in less time than A
- d. A will reach the ground in less time than B

Q.10 A body is projected horizontally from a point above the ground. The motion of body is defined as $x=2t$ and $y=2t^2$

where x and y are horizontal and vertical displacement respectively at time t . Which one of the following is true

- a. The trajectory of the body is a parabola
- b. The trajectory of the body is a straight line
- c. the velocity vector at point t is $2\mathbf{i}+4t\mathbf{j}$
- d. the acceleration vector at time t is $4\mathbf{j}$

Answer Key

1 E

2 B

3 d

Solution:

From the question, we know that,

$$\vec{v}_{DC} = \vec{v}_D - \vec{v}_C = 20\hat{j} \quad \dots\dots (1)$$

$$\vec{v}_{BC} = \vec{v}_B - \vec{v}_C = 20\hat{i} \quad \dots\dots (2)$$

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -20\hat{j} \quad \dots\dots (3)$$

Equation (1) – (2) + (3) gives:

$$\vec{v}_D - \vec{v}_A = -20\hat{j}$$

$$\text{or } \vec{v}_{DA} = -20\hat{j}$$

That is, \vec{v}_{DA} is 20 m/s towards west.

Thus from the above observation we conclude that, option (d) is correct

4 B

5 d

6 D

7 a

$$\text{Distance} = \frac{1}{2} at^2$$

$$490 \text{ m} = 9.8 t^2 / 2$$

$$T = \text{SQRT} ((490 / 9.8) *2) = 10 \text{ seconds}$$

8 A

9 A

10 a

working in ref frame attached to 2,
velocity of 1 wrt 2= v_1+v_2

let length of train be x metres each.

$$\text{therefore time taken to cross each other} = 2x/v_1+v_2=3$$

$$\Rightarrow 2x = 3v_1+3v_2 \text{-----}(1)$$

Case 2:

$$\text{velocity of 1} = v_1+50v_1/100=3v_1/2$$

$$\text{now, time taken to cross each other} = \frac{2x}{3v_1/2 - v_2}$$

$$\text{-----} = 2.$$

$$3v_1/2 - v_2$$

$$\Rightarrow 2x = 2.5v_2 + 7.5v_1/2 \text{-----}(2)$$

equating (1) and (2)

$$3v_1+3v_2 = 2.5v_2+7.5v_1/2$$

$$\Rightarrow 1.5v_1/2 = v_2 \text{-----}(3)$$

now if they move in same direction:

$$\text{velocity of 1 wrt 2} = v_1-v_2$$

$$\text{time taken to cross each other} = 2x/v_1-v_2 = 2x/v_1-0.75v_1 \text{ (from 3)}$$

$$= 2x/0.25v_1$$

$$= 3v_1+3 v_2/ 0.25v_1 \text{ (from 1)}$$

$$= 3v_1+ 0.75v_1/0.25v_1 \text{ (from 3)}$$

$$= 3.75v_1/0.25v_1 = 15 \text{ sec}$$

a

for the 1st ball
distance moved in 3 seconds,

$$s=ut+(1/2)gt^2$$

for a freely falling body $u=0$

$$s=(1/2)gt^2$$

$$s=(1/2)\times 10\times 3^2$$

$$s=45\text{m}$$

for the 2nd ball

when real time = 3s the body has been let into free fall for only 2s as the 2nd ball was dropped one second after the first ball

hence $t=2\text{s}$

$$s=ut + (1/2)gt^2$$

$$s=(1/2)\times 10\times 2^2$$

$$s=20\text{m}$$

distance between them $(45-20)\text{m} = 25\text{m}$

11

12

d

mass of 'A' is M_a

mass of 'B' is M_b

both of them undergo an acceleration of 'g' as it is falling from a height
initial velocity of both is zero as it is dropped from rest.

From 2nd equation of motion.

$$S = 1/2 a t^2 \quad [\text{since } u = 0, \text{ it is not taken. }]$$

$$a = 1/2g t_1^2$$

$$b = 1/2g t_2^2$$

$$a/b = t_1^2 / t_2^2$$

$$t_1 / t_2 = \sqrt{a/b}$$

this the the ratio of times taken by M_a and M_b

13

d

$$= ax^2 + bx$$

differentiate w.r.t t

$$1 = 2ax \, dx/dt + b \, dx/dt$$

$$1 = (2ax+b)dx/dt$$

$$dx/dt = 1/(2ax+b)$$

again differentiate

$$d^2x/dt^2 = -1/(2ax+b)^2 (2a \, dx/dt)$$

$$a = -1/(2ax+b)^2 * 2a/(2ax+b)$$

$$a = -2a/(2ax+b)^3$$

14

a

Let the half time intervals be t

$$\text{Therefore } 60 = 80t + 40t$$

$$t=1/2$$

$$\text{total time} = 1/2 * 2 = 1\text{hr}$$

$$v_{av} = \text{Total distance} / \text{Total time} = 60 / 1 = 60 \text{ km/h}$$

15

c

$$x \propto t^{1/2}$$

By differentiation ,

$$v \propto 1/t^{1/2}$$

Again by differentiation,

$$a \propto 1/t^{3/2}$$

Therefore $a \propto v^3$

16 d

b

$$T = \sqrt{(2h/g)}$$

$$= \sqrt{(2 \times 19.6)/9.8}$$

17 = 2 cm

18 c

$$s = 0 \times 4 + \frac{1}{2} \times 9.8 \times 4^2$$

$$\text{Hence, } s = 78.4 \text{ m}$$

19 b

$$h_1/h_2 = t_1^2/t_2^2 = \frac{1}{4} = 0.25$$

20 a

$$x = t^3 - 4t^2 + 3t$$

$$v = 3t^2 - 8t + 3$$

$$a = 6t - 8$$

$$t = (a+8)/6$$

$$A/q, \quad x=0$$

$$0 = t^3 - 4t^2 + 3t$$

$$0 = t(t-3)(t-1)$$

$$t = 0$$

$$(a+8)/6 = 0$$

$$a = -8$$

Similarly, $t=3$

$$(a+8)/6 = 3$$

$$a = 10$$

$$= 3$$

$$(a+8)/6 = 1$$

$$a = -2$$

21 D

22 b

Time of journey (free fall) = $t = n$ seconds

Height of free fall = H meters ;

initial velocity = 0 ; acceleration due to gravity = $g = 9.8 \text{ m/s}^2$

$Hn = (1/2) g t^2 = (1/2) g n^2 =$ distance travelled in n seconds

$H(n-1) = (1/2) g (n - 1)^2 =$ distance travelled in $n - 1$ seconds

Distance travelled in the n th (last) second = $h = Hn - H(n-1)$

$$= (1/2)g \{n^2 - (n - 1)^2\} = (1/2) g (2n - 1)$$

As per the question : $h = (1/2) Hn$

$$(1/2) g (2n - 1) = (1/4) g n^2$$

$$\Rightarrow 2n - 1 = n^2/2$$

$$\Rightarrow n^2 = 4n - 2$$

$$\Rightarrow n^2 - 4n + 2 = 0$$

$$\Rightarrow (n - 2)^2 - 2 = 0$$

$$\Rightarrow (n - 2)^2 = 2$$

$$\Rightarrow n - 2 = \pm \sqrt{2}$$

$$\Rightarrow n = 2 \pm \sqrt{2}$$

$$\Rightarrow n = (2 + \sqrt{2}) \text{ or } (2 - \sqrt{2})$$

$n = 2 - \sqrt{2}$, being less than 1, is rejected.

Time of journey = $n = 2 + \sqrt{2}$ seconds

23 A

24 D

25 b

When angle of projection is same than $u_2 = \sqrt{3} u_1$

$$T_2 = \sqrt{3} T$$

26 a

27 E

28 A

29 B

30 c

Resultant velocity of cars is $\sqrt{u_1^2 + u_2^2 + u_1 u_2 \cos \theta}$

$$t = s / \sqrt{u_1^2 + u_2^2 + u_1 u_2 \cos \theta}$$

ANSWER

- Q.1 (d) This is so because bus is changing its direction of motion
- Q.2 b,c
- Q.3 a,b,c,d
- Q.4 a,b,
- Q.5 a,b
- Q.6 b,c
- Q.7 a,b,c
- Q.8 a,b,d
- Q.9 a,b
- Q.10 a,c,d

UNIT: III Laws of Motion Important Formulae

$$F = ma$$

$$F = \frac{dp}{dt}$$

$$p = mv$$

For atwood machine

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g \quad T = \frac{2m_1 m_2}{(m_1 + m_2)} g$$

Friction

$$f_s = \mu_s N \quad f_L = \mu_L N \quad f_k = \mu_k N$$

Angle of friction and angle of repose

$$\tan \theta = \frac{f}{N} = \mu$$

Safe speed on levelled road and banked road

$$v = \sqrt{\mu r g} \quad v = \sqrt{r g \left(\frac{\tan \theta + \mu}{1 - \mu \tan \theta} \right)}$$

Impulse:

of a force is the product of the force and the time interval over which it acts. Impulse is a vector quantity. The impulse (J) delivered by a changing force is expressed as

$$J = F_{av} t = \Delta p$$

MCQ (Single option correct questions)

1. A 10N force is applied on a body to produce in it acceleration an acceleration of 1 m/s^2

The mass of the body is

- (a) 15kg
- (b) 20kg
- (c) 10kg
- (d) 5kg

2. A force of 2N is applied on a particle for 2 sec, the change in momentum will be-

- (a) 4 Ns
- (b) -4 Ns
- (c) 2 Ns
- (d) 3 Ns

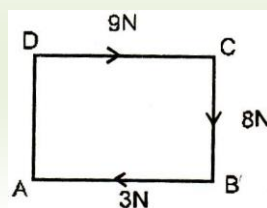
3. A cricketer catches a ball of mass 150g in 0.1s moving with a speed 20m/s then he experiences a force of

- a) 300N
- b) 30N
- c) 3N
- d) 0.3N

4. Physical independence of force is consequences of

- a) Third law of motion
- b) Second law of motion
- c) first law of motion
- d) all of these law

5. A stone is dropped from a height h . It hits the ground with the certain momentum p . If the same stone is dropped from the height 100% more than the previous height, the momentum when it hits the ground will change by
- 68%
 - 41%
 - 200%
 - 100%
6. A balloon of mass m is descending down with an acceleration a (where $a < g$) how much mass should be removed from it so that it start moving up with an acceleration a ?
- $2ma/g+a$
 - $Ma/g+a$
 - $2ma/g-a$
 - $Ma/g-a$
7. On the horizontal surface of a truck a block of mass 1 kg is placed ($\mu=0.6$) and truck is moving with acceleration 5m/s^2 then the frictional force on the block will be
- 5N
 - 6N
 - 5.88N
 - 8N
8. A body whose mass 6 kg is acted upon by two forces $8\hat{i} + 10\hat{j}$ and $4\hat{i} + 8\hat{j}$ N. The acceleration produced will be – (in ms^{-2})
- $3\hat{i} + 2\hat{j}$
 - $12\hat{i} + 18\hat{j}$
 - $\frac{1}{3}(\hat{i} + \hat{j})$
 - $2\hat{i} + 3\hat{j}$
9. Two identical billiard balls strike a rigid wall with the same speed but at different angles, and get reflected without any change in speed, as shown in Fig. (a) What is (i) the direction of the force on the wall due to each ball? (ii) the ratio of the magnitudes of impulses imparted to the balls by the wall?
- zero
 - 1.2
 - 3.2
 - 1.2
10. ABCD is a rectangle forces of 9N, 8N, 3N act along the lines DC, CB and BA, respectively, in the directions indicated by the order of the letters. Then the resultant force is



(a) 8 N

(b) 5 N

(c) 20 N

(d) 10 N

11. . A body is sliding down an inclined plane having coefficient of friction 0.5. If the normal reaction is twice that of the resultant downward force along the incline, the angle between the inclined plane and the horizontal is

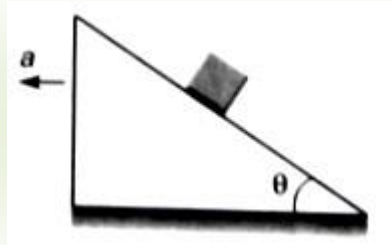
(a) 15°

(b) 30°

(c) 45°

(d) 60°

12. A block of mass m is resting on a wedge of angle θ as shown in the figure. With what minimum acceleration a should be wedge move so that the mass m falls freely?



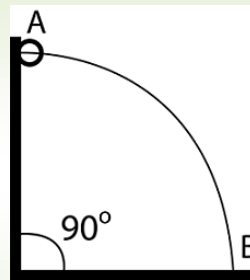
(a) g

(b) $g \cos \theta$

(c) $g \cot \theta$

(d) $g \tan \theta$

13. A wire, which passes through a hole in a small bead, is bent in the form of a quarter of a circle. The wire is led vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is:



(a) always radially outwards

(b) always radially inwards

(c) radially outwards initially and radially inwards later

(d) radially inwards initially and radially outwards later

14. A rocket of initial mass m_0 , moving with velocity V , discharges a jet of gases of mean density ρ and effective area A . The minimum value of v of fuel gas, which enables the rocket to rise vertically above is nearly :

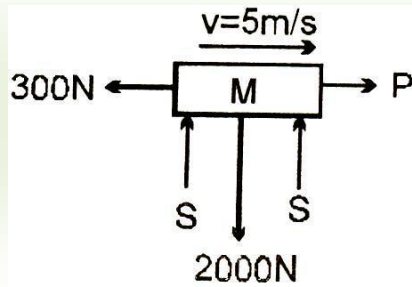
(a) $\sqrt{\frac{\rho g}{m_0 A}}$

(b) $\sqrt{\frac{\rho g A}{m_0}}$

(c) $\sqrt{\frac{m_0 g}{\rho A}}$

(d) $\sqrt{\frac{m_0 g A}{\rho}}$

15. The forces acting on an object are shown in the fig. If the body moves horizontally at a constant speed of 5 m/s, then the values of the forces P and S are , respectively –



- (a) 0 N, 0 N (b) 300 N, 200 N (c) 300 N, 1000 N (d) 2000 N, 300 N

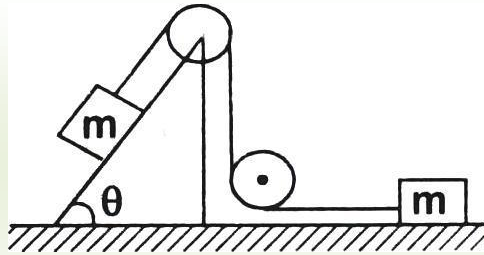
16. A block of metal weighing 2kg is resting on a frictionless plane. It is struck by a jet releasing water at a rate of 1kg/s and at a speed of 5m/s. The initial acceleration of the block will be

- (a) 2.5 ms^{-2} (b) 5 ms^{-2} (c) 0.4 ms^{-2} (d) 0

17. A ball is moving with a velocity v strikes a wall moving towards the ball with velocity u . An elastic impact last for t sec. Then the mean elastic force acting on the ball is

- (a) $\frac{2m(v+u)}{t}$ (b) $\frac{2m(v+2u)}{t}$ (c) $\frac{m(2v+u)}{t}$ (d) $\frac{m(v+2u)}{t}$

18. For the system shown in the figure, the pulleys are light and frictionless. The tension in the string will be



- (a) $\frac{2}{3} mg \sin \theta$ (b) $\frac{3}{2} mg \sin \theta$ (c) $\frac{1}{2} mg \sin \theta$ (d) $2mg \sin \theta$

19. An open knife edge of mass M is dropped from a height h on a wooden floor. If the blade penetrates S into the wood, the average resistance offered by the wood to the blade is

- (a) Mg (b) $Mg \left(1 + \frac{h}{s}\right)$ (c) $Mg \left(1 - \frac{h}{s}\right)$ (d) $Mg \left(1 + \frac{h}{s}\right)^2$

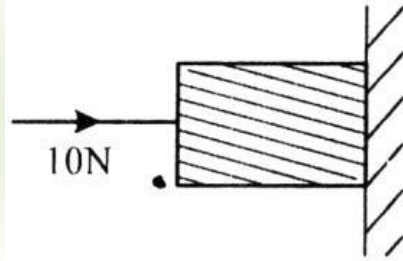
20. A 25 kg lift is supported by a cable. The acceleration of the lift when the tension in the cable is 175N, will be

- (a) -2.8 ms^{-2} (b) 16.8 ms^{-2} (c) -9.8 ms^{-2} (d) 14 ms^{-2}

21. A particle is observed from two frames s_1 and s_2 . The frame s_2 moves with respect to s_1 with an acceleration a . Let F_1 and F_2 be the pseudo forces on the particle when seen from s_1 and s_2 respectively. Which of the following are not possible ?

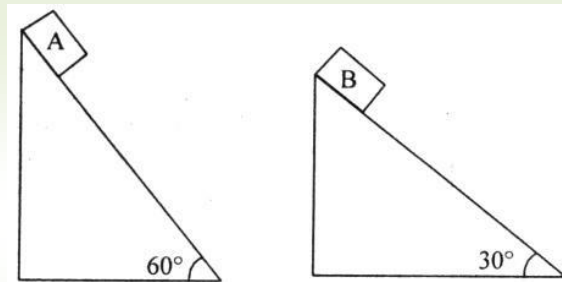
- (a) $F_1 = 0, F_2 \neq 0$ (b) $F_1 \neq 0, F_2 \neq 0$ (c) $F_1 \neq 0, F_2 = 0$ (d) $F_1 = 0, F_2 = 0$

22. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is



- (a) 20 N (b) 50 N (c) 100N (d) 2N

23. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ?



- (a) 4.9 ms^{-2} in horizontal direction (b) 9.8 ms^{-2} in vertical direction (c) Zero (d) 4.9 ms^{-2} in vertical direction

24. A curve has a radius of 50 meters and a banking angle of 15° . What is the ideal, or critical, speed (the speed for which no friction is required between the car's tires and the surface) for a car on this curve?

- (a) 490 m/s (b) 132 m/s (c) 11 m/s (d) 22 m/s

25. A 1200 kg automobile rounds a level curve of radius 200 m, on a unbanked road with a velocity of 72 km/hr. What is the minimum co-efficient of friction between the tyres and road in order that the automobile may not skid. ($g = 10 \text{ m/s}^2$)

- (a) 0.2 (b) 2.0 (c) 0.02 (d) 20.0

26. Speeds of two identical cars are u and $4u$ at the specific instant. The ratio of the respective distances in which the two cars are stopped from that instant is

- (a) 1 :1 (b) 1 :4 (c) 1:8 (d) 1:16.

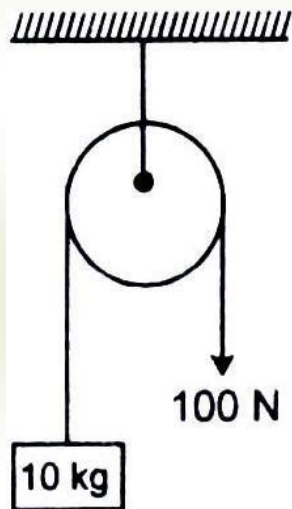
27. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is

- (a) $mg(\sqrt{2} + 1)$ (b) $mg\sqrt{2}$ (c) $\frac{mg}{\sqrt{2}}$ (d) $mg(\sqrt{2} - 1)$

28. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k . The block are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force of the block of mass m

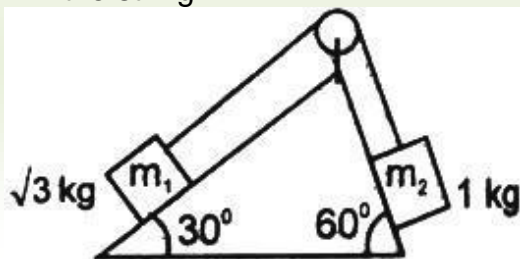
- (a) $\frac{MF}{(m+M)}$ (b) $\frac{mF}{M}$ (c) $\frac{(M+m)F}{m}$ (d) $\frac{mF}{(m+M)}$

29. One end of string which passes through pulley and connected to 10 kg mass at other end is pulled by 100 N force. Find out the acceleration of 10 kg mass, ($g = 9.8 \text{ m/s}^2$)



- (a) 0.2 m/s^2 (b) 2.0 m/s^2 (c) 0.02 m/s^2 (d) 20.0 m/s^2

30. Two blocks m_1 and m_2 are placed on a smooth inclined plane as shown in figure. If they are released from rest. Find tension in the string



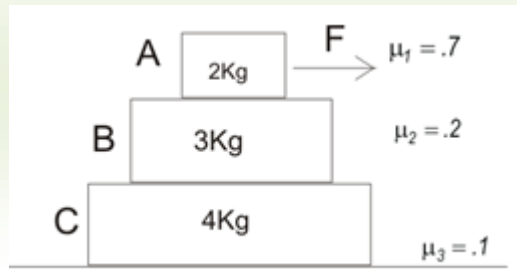
- (a) $\frac{3}{2}g$ (b) $\frac{\sqrt{3}}{2}g$ (c) $\frac{\sqrt{3}}{2}g$ (d) $\frac{3}{\sqrt{2}}g$

Answer Key for MCQ

1	c	7	a	13	d	19	b	25	a
2	a	8	d	14	c	20	a	26	d
3	b	9	d	15	c	21	b	27	d
4	c	10	d	16	a	22	b	28	d
5	b	11	a	17	a	23	a	29	a
6	a	12	c	18	c	24	c	30	b

MCQ with more than one correct option

1 A, B, C are the objects as shown above in the figure. A, B, C are 2, 3 and 4 Kg respectively. Coefficient of friction between the blocks are given above in the figure



for $F = 15 \text{ N}$

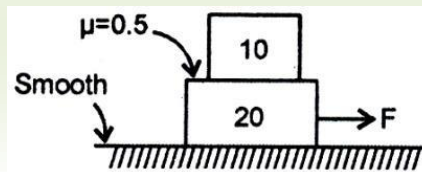
- (a) there will be relative motion between A & B
- (b) there will be relative motion between B & C
- (c) there will be relative motion between C & surface
- (d) none of the above

Answer Key

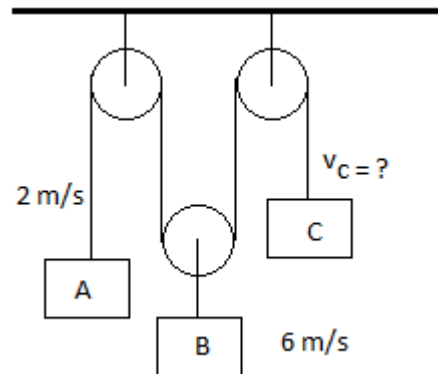
1. (b),(c)

5 Marks Questions

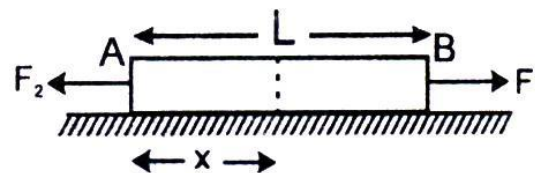
1. Initially the system is at rest, find out minimum value of F for which sliding starts between the two blocks.



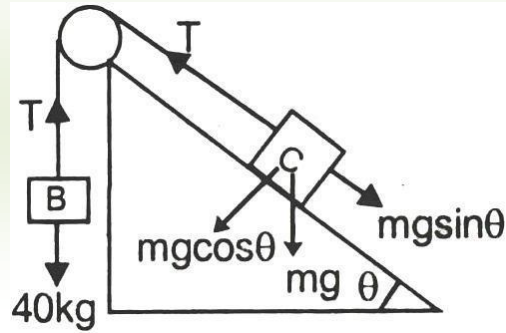
2. Find speed of block C



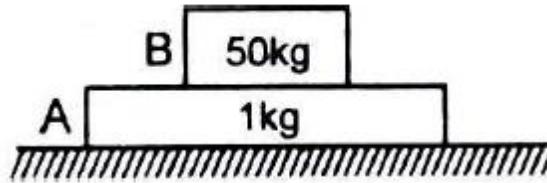
3. Two forces F_1 and $F_2 (> F_1)$ are applied at the free ends of uniform rod kept on a horizontal frictionless surface. Find tension in rod at a distance x from end 'A',



4. A body of mass 50kg resting on a smooth inclined plane is connected by massless inextensible string passing over a smooth pulley at the top of the inclined plane to another mass 40kg as shown. The distance through which 50kg mass rises in 4 sec. will be- (The angle of the inclined plane is 30°)



5. A block of mass 50 kg is kept on another block of mass 1 kg as shown in fig. a horizontal force of 10 N is applied on the block. (All surfaces are smooth) Find: (a) Acceleration of block A and B (b) Force exerted by B on A. (take $g=10\text{ms}^{-2}$)



Solutions:

1. At just sliding condition limiting friction is acting.



$$F - 50 - 20 a \dots (1)$$

$$f = 10 a \dots (2)$$

$$50 = 10 a \quad a = 5 \text{ m/s}^2$$

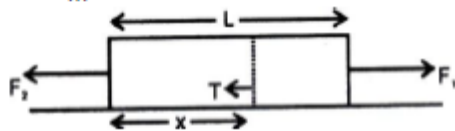
$$\text{Hence } F = 50 + 20 \times 5 = 150 \text{ N}$$

$$F_{\text{min}} = 150 \text{ N}$$

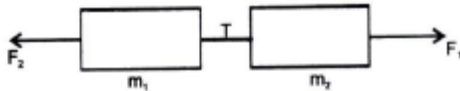
3.

$$\text{Ans. } T = F_2 - \frac{F_2 - F_1}{L} \cdot x$$

$$\text{Sol. } a = \frac{F_2 - F_1}{m}$$



$$T - F_1 = m_2 a$$



$$\Rightarrow T - F_1 = \frac{m}{L} (L-x) \frac{F_2 - F_1}{m} \quad (m_2 = \frac{m}{L} (L-x))$$

$$\Rightarrow T = F_1 + \left(1 - \frac{x}{L}\right) (F_2 - F_1)$$

$$= F_1 + F_2 - F_1 - \frac{x}{L} (F_2 - F_1)$$

$$= F_2 - \frac{x}{L} (F_2 - F_1)$$

4.

The tension is same in two segments

For B the equation is

$$(40 \times 9.8 - T) = 40a \quad \dots\dots(i)$$

For C the equation is

$$\left(T - 50 \times 9.8 \times \frac{1}{2}\right) = 50a \quad \dots\dots(ii)$$

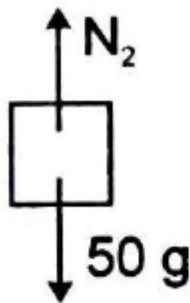
From equation (i) and (ii)

$a = 1.63 \text{ m/s}^2$ distance of rise

$$S = \frac{1}{2}at^2 = \frac{1}{2} \times 1.63 \times 42 = 13.04 \text{ m}$$

5.

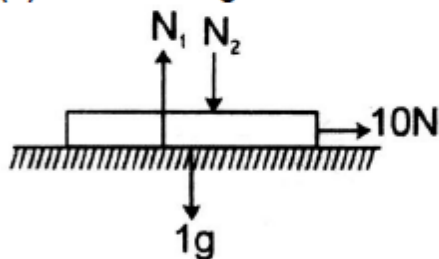
(a) F.B.D. of 50 kg



$$N_2 = 50g = 500 \text{ N}$$

Along horizontal direction, there is no force $a_B = 0$

(b) F.B.D. of 1 kg block:



along horizontal direction

$$10 = 1a_A$$

$$a_A = 10 \text{ m/s}^2$$

along vertical direction

$$\therefore N_1 = N_2 + 1g$$

$$= 500 + 10 = 510 \text{ N}$$

MCQ WITH MORE THAN ONE OPTION CORRECT

Q.1 This question contains statement-1 (Assertion) and Statement-2 (Reason).

Statement-1

A bus moving due north take a turn and starts moving towards east with same speed. There will be no change in the velocity of the bus.

Statement-2

Velocity is a vector quantity.

- (A) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1.
- (B) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1.
- (C) Statement-1 is true, Statement-2 is false.
- (D) Statement-1 is false, Statement-2 is true.

Q.2 The velocity, displacement, acceleration of a particle in one dimensional motion is given as v_1, x_1, a_1 at $t=t_0$, v_2, x_2, a_2 at $t=t_0 + \Delta t$ which of the following is correct

- a) $x_2 = x_1 + v_1 \Delta t + \frac{1}{2} a_1 \Delta t^2$
- b) $x_2 - x_1 = v_{avg} \Delta t$
- c) $v_2 - v_1 = a_{avg} \Delta t$
- d) $x_2 = x_1 + v_{avg} \Delta t + \frac{1}{2} a_{avg} \Delta t^2$

Q.3 The displacement time equation for a particle in linear motion is given as

$$x = \frac{a}{b} (1 - e^{-bt})$$

which of the following option is correct

- a) The velocity and acceleration of the particle at $t=0$ is a and $-ab$ respectively
- b) The velocity will be decreasing as the time increases
- c) The displacement of the particle will fall between

$$0 \leq x \leq \frac{a}{b}$$

- d) The maximum acceleration in the motion is $-ab$

Q.4 Which one is wrong for a body having uniform circular motion?

- a). Speed of the body is constant
- b). Acceleration is directed towards the centre
- c). Velocity and Acceleration vector are having an angle 45

d). none of the above

- Q.5 A body moves along a semicircular track of Radius R. Which of the following statement is true
- a. Displacement of the body is $2R$
 - b. Distance travelled by the body is πR
 - c. Displacement of the body is πR
 - d. none of the above

- Q.6 A particle is going moving along x-axis. Which of the following statement is false
- a. At time t_1 $(dx/dt)_{t=t_1}=0$, then $(d^2x/dt^2)_{t=t_1}=0$
 - b. At time t_1 $(dx/dt)_{t=t_1} < 0$ then the particle is directed towards origin
 - c. If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.
 - d. At time t_1 $(d^2x/dt^2)_{t=t_1} < 0$ then the particle is directed towards origin
- Q.7

A particle starts at time $t=0$ from $x=0$ along the positive x-axis with constant speed v . After time t , it return back towards the origin with the speed $2v$ and reaches the origin in $t/2$ sec. Which of the following is true for the whole process

- a. Average velocity is zero for the whole process
 - b. Average speed is $4/3v$ for the whole process
 - c. Displacement at time t is equal to vt
 - d. Displacement at time $3t/2$ is $2vt$
- Q.8 The range of the projectile depends upon
- a. Angle of the projection
 - b. Acceleration due to gravity
 - c. Mass of the projectile
 - d. magnitude of the velocity of projection
- Q.9 Two bullets A and B are fired horizontally with speed v and $2v$ respectively. Which of the following is true
- a. Both will reach the ground in same time
 - b. Bullet with speed $2v$ will cover more horizontal distance on the ground
 - c. B will reach the ground in less time than A
 - d. A will reach the ground in less time than B

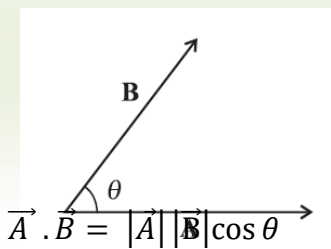
- Q.10 A body is projected horizontally from a point above the ground. The motion of body is defined as $x=2t$ and $y=2t^2$ where x and y are horizontal and vertical displacement respectively at time t . Which one of the following is true
- a. The trajectory of the body is a parabola
 - b. The trajectory of the body is a straight line
 - c. the velocity vector at point t is $2\mathbf{i}+4t\mathbf{j}$
 - d. the acceleration vector at time t is $4\mathbf{j}$

ANSWER

- Q.1 (d) This is so because bus is changing its direction of motion
- Q.2 b,c
- Q.3 a,b,c,d
- Q.4 a,b,
- Q.5 a,b
- Q.6 b,c
- Q.7 a,b,c
- Q.8 a,b,d
- Q.9 a,b
- Q.10 a,c,d

UNIT- IV -WORK, ENERGY AND POWER

Scalar Product

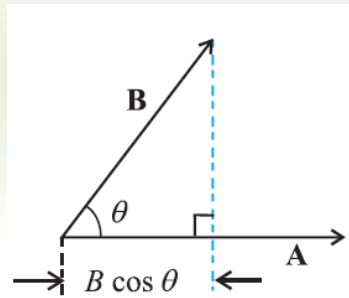


$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\vec{i} \cdot \vec{i} = |\vec{i}| |\vec{i}| \cos 0$$

$$\vec{i} \cdot \vec{i} = 1, \vec{j} \cdot \vec{j} = 1, \vec{k} \cdot \vec{k} = 1$$

$$\vec{i} \cdot \vec{j} = \vec{i} \cdot \vec{k} = \vec{j} \cdot \vec{k} = 1 \times 1 \times \cos \frac{\pi}{2} = 0$$

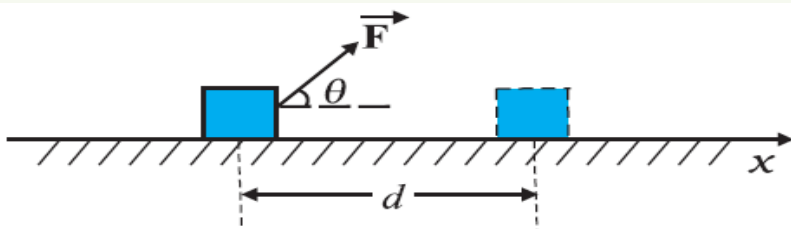


WORK-ENERGY THEOREM

The change in kinetic energy of a particle is equal to the work done on it by the net force.

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = W = \vec{F} \cdot \vec{d}$$

WORK



The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement.

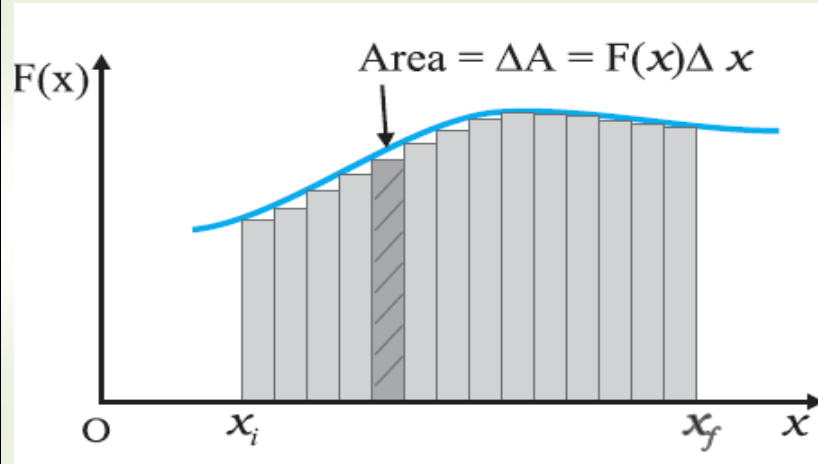
$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta = d(F \cos \theta)$$

KINETIC ENERGY

If an object of mass m has velocity \vec{v} , its kinetic energy K is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\vec{v} \cdot \vec{v}$$

WORK DONE BY A VARIABLE FORCE



$$w = \int_{x_i}^{x_f} f(x)dx$$

WORK-ENERGY THEOREM FOR A VARIABLE FORCE

$$w = k_f - k_i = \int_{x_i}^{x_f} f(x)dx$$

THE CONCEPT OF POTENTIAL ENERGY

The notion of potential energy is applicable only to the class of forces where work done against the force gets 'stored up' as energy. When external constraints are removed, it manifests itself as kinetic energy. Mathematically, the potential energy $V(x)$ is defined if the force $F(x)$ can be written as

$$f(x) = -\frac{dV}{dx}$$

THE CONSERVATION OF MECHANICAL ENERGY

The sum of the kinetic and potential energies of the body is a constant.

$$k_i + V_i = k_f + V_f = E$$

* A force $F(x)$ is conservative if it can be derived from a scalar quantity $V(x)$ by the relation given above.

* The work done by the conservative force depends only on the end points. This can be seen from the relation,

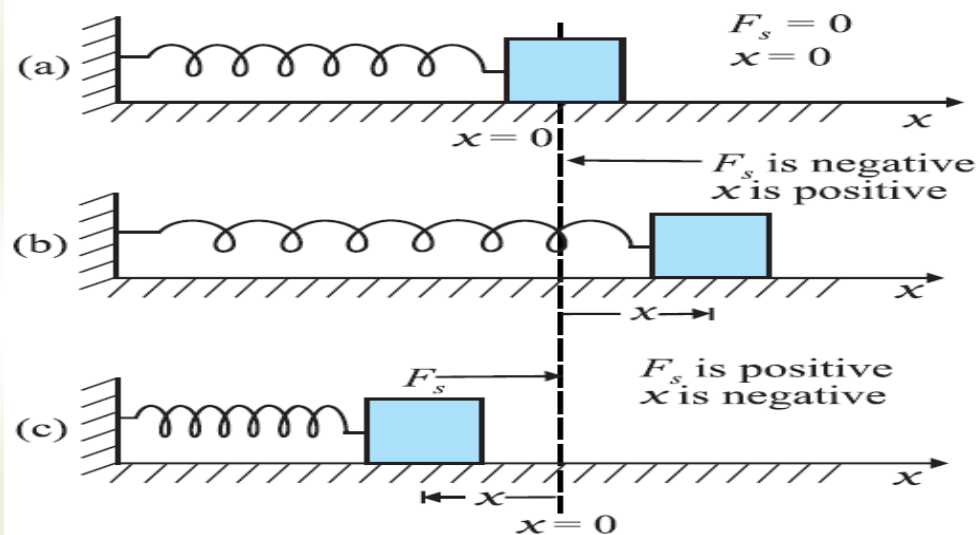
$$w = k_f - k_i = V(x_i) - V(x_f)$$

which depends on the end points.

* A third definition states that the work done by this force in a closed path is zero.

The total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.

THE POTENTIAL ENERGY OF A SPRING



$$U = \frac{1}{2} kx_m^2$$

$$\frac{1}{2} kx_m^2 = \frac{1}{2} kx^2 + \frac{1}{2} mv^2$$

$$\frac{1}{2} kx_m^2 = \frac{1}{2} mv_m^2$$

POWER

Power is defined as the time rate at which work is done or energy is transferred.

The instantaneous power is defined as the limiting value of the average power as time interval approaches zero,

$$P = \frac{dW}{dt}, \text{ but } W = \vec{F} \cdot \vec{s}$$

$$P = \frac{d(\vec{F} \cdot \vec{s})}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

COLLISIONS

Elastic collision in one dimension between two balls of masses m_1 and m_2 moving with velocities u_1 and u_2 where $u_1 > u_2$ than final velocities

$$v_1 = \frac{(m_1 - m_2)}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2$$

$$v_2 = \frac{(m_2 - m_1)}{m_1 + m_2} u_2 + \frac{2m_1}{m_1 + m_2} u_1$$

FORMULAE OF THE CHAPTER AT GLANCE

1. $W = \mathbf{F} \cdot \mathbf{S} = F S \cos \Phi$ where Φ is the smaller angle between F and S
2. If a body of mass m is raised through height h , then $W = mgh$

3. If a body moves up a plane inclined at an angle Φ with constant speed, then
 $W = mg \sin\Phi \times s$
4. Kinetic energy, $K = \frac{1}{2} mv^2$
5. According to work – energy theorem,
 $W = K_f - K_i = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$
6. Gravitational potential energy, $U = mgh$
7. For conservative force, $F = -dU/dx$
8. When work is done only by conservative forces only, mechanical energy is conserved,
 $K + U = \text{constant}$
9. According to Hook's law, $F = -kx$
10. Force constant, $k = F/x$
11. Work done on a spring or P.E. of a spring stretched through distance x , $W = U = \frac{1}{2} kx^2$
12. Power = work/time or $P = W/t$
13. Also $P = \mathbf{F} \cdot \mathbf{V}$ when $\Phi = 0$, $P = FV$
14. According to Einstein, energy equivalent of mass m is $E = mc^2$
15. Linear momentum is conserved both in elastic and inelastic collisions.
 $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$
16. Kinetic energy is conserved in elastic collision.
 $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$
17. In one dimensional elastic collision, velocities after the collision are given by
 $v_1 = \left\{ \frac{m_1 - m_2}{m_1 + m_2} \right\} \cdot u_1 + \left\{ \frac{2 m_2}{m_1 + m_2} \right\} \cdot u_2$
 $v_2 = \left\{ \frac{m_2 - m_1}{m_1 + m_2} \right\} \cdot u_2 + \left\{ \frac{2 m_1}{m_1 + m_2} \right\} \cdot u_1$

Special cases:

- i) When $m_1 = m_2 = m$ then $v_1 = u_2$ and $v_2 = u_1$
- ii) When $m_1 = m_2 = m$ and $u_2 = 0$ then $v_1 = 0$ and $v_2 = u_1$
- iii) When $m_1 \ll m_2$ and $u_2 = 0$ then $v_1 = -u_1$ and $v_2 = 0$
- iv) When $m_1 \gg m_2$ and $u_2 = 0$ then $v_1 = u_1$ and $v_2 = 2u_1$

18. In one dimensional elastic collision,

$$U_1 - U_2 = V_2 - V_1 \text{ i.e relative velocity of approach} = \text{relative velocity of separation}$$

19. Coefficient of restitution or coefficient of resilience

$$e = - (V_1 - V_2) / (U_1 - U_2)$$

- i) For perfectly elastic collision , $e = 1$
- ii) For an inelastic collision $0 < e < 1$ for glass balls $e = 0.95$ and lead balls $e = 0.20$
- iii) For perfectly inelastic collision $e = 0$
- iv) For super elastic collision $e > 1$ i.e kinetic energy increases

20. For ball rebounding from a floor, $e = v/u$

21. $1 \text{ J} = 1 \text{ Nm}$ and joule = 10^7 erg

22. Relation between K.E. and linear momentum

$$P = (2mK)^{1/2}$$

23. Einstein's mass energy equivalence:

$$E = mc^2 \text{ and } 1 \text{ amu} = 1u = 1.661 \times 10^{-27} \text{ kg} = 931 \text{ MeV}$$

24. When a body moves with a velocity 'v' comparable to the velocity of light 'c'

$$m = m_0 / \{ 1 - (v^2 / c^2) \}^{1/2} \text{ where } m_0 \text{ is its rest mass}$$

25. Average power $P_{av} = W/t$ 1 Watt = 1 J/s

26. $1 \text{ kW} = 1000 \text{ W}$ $1 \text{ hp} = 746 \text{ W}$ $1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT OPTION

1. A light and a heavy body have equal momentum. Which one has greater K.E.?
- (a) The light body
 - (b) Both have equal K.E.
 - (c) The heavy body
 - (d) Data given is incomplete.

2. A bullet weighing 5 gm and moving with a velocity 600 m/s strikes a 5 kg block of ice resting on a frictionless surface. The speed of the block after the collision is
 (a) 6 cm/s (b) 60 cm/s
 (c) 6 m/s (d) 0.6 cm/s.
3. Two masses of 1 gm and of 4 gm are moving with equal linear momentum. The ratio of their kinetic energies is
 (a) 4:1 (b) $\sqrt{2} : 1$
 (c) 1:2 (d) 1:16
4. If the linear momentum is increased by 50%, than K.E. will be increased by :
 (a) 50% (b) 100%
 (c) 125% (d) 25%
5. A long spring is stretched by 2 cm. Its potential energy is V. If the spring is stretched by 10 cm, its potential energy would be
 (a) V/25 (b) V/5
 (c) 5V (d) 25V
6. A bullet of mass a and velocity b is fired into a large block of mass c. The final velocity of the system is
 (a) $\frac{cb}{a+b}$ (b) $\frac{ab}{a+c}$
 (c) $\frac{(a+b)a}{c}$ (d) $\frac{(a+c)b}{a}$
7. An engine develops 10 kW of power. How much time will it take to lift a mass of 200 kg to a height of 40 m ($g = 10 \text{ ms}^{-2}$)
 (a) 4s (b) 5s
 (c) 8s (d) 10s
8. A body constrained to move in y-direction is subjected to a force given by $\vec{F} = (-2\hat{i} + 15\hat{j} + 6\hat{k})N$. What is the work done by this force in moving the body through a distance of 10m along y-axis?
 (a) 190J (b) 160J
 (c) 150J (d) 20J
9. If $\vec{A} = 5\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{B} = 2\hat{i} + 2\hat{j} - c\hat{k}$ are perpendicular vectors, the value of c is
 (a) -2 (b) 8
 (c) -7 (d) -8
10. Angle that the vector $\vec{A} = 2\hat{i} + 3\hat{j}$ makes with y-axis is
 (a) $\tan^{-1} \frac{3}{2}$ (b) $\tan^{-1} \frac{2}{3}$
 (c) $\tan^{-1} \frac{3}{5}$ (d) $\tan^{-1} \frac{3}{\sqrt{13}}$
11. Two Vectors \vec{A} and \vec{B} are such that $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$. Then angle between the vectors \vec{A} and \vec{B} is
 (a) 0° (b) 60°
 (c) 90° (d) 180°
12. The angle between the two vectors $-2\hat{i} + 3\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$ is
 (a) 0° (b) 90°

(c) 180°

(d) None

13. If $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{B} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular vectors, the positive value of c is

(a) 3

(b) 2

(c) 1

(d) 0

14. What can be the angle between $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$?

(a) 0° only

(b) 60° only

(c) 90° only

(d) between 0° to 180°

15. The work done is represented by

(a) $W = \vec{F} \cdot \vec{s}$

(b) $W = -\vec{F} \cdot \vec{s}$

(c) $W = \vec{s} \cdot \vec{F}$

(d) $W = -\vec{s} \cdot \vec{F}$

16. A particle moves along the x-axis from $x=0$ to $x=5\text{m}$ under the influence of a force given by $F = 7 - 2x + 3x^2$. Work done in the process is

(a) 70

(b) 270

(c) 35

(d) 135

17. The K.E. acquired by a mass m in travelling a certain distance d , starting from rest, under the action of a constant force is directly proportional

(a) m

(b) \sqrt{m}

(c) $\frac{1}{\sqrt{m}}$

(d) none of the above

18. Consider a car moving along a straight horizontal road with a speed of 72 Km/h. If the coefficient of static friction between road and tyres is 0.5, the shortest distance in which the car can be stopped is

(a) 30m

(b) 40m

(c) 72m

(d) 20m

19. Out of a pair of identical springs of spring constant 240 N/m, one is compressed by 10 cm and the other is stretched by 10 cm. The difference in potential energy stored in the two springs is

(a) Zero

(b) 4J

(c) 12J

(d) 1.2J

20. In which case does the potential energy decreases?

(a) on compressing a spring

(b) on stretching a spring

(c) on moving a body against gravitational pull

(d) on raising of an air bubble in water

21. Two bodies with kinetic energies in ratio of 4 : 1 are moving with equal linear momentum. The ratio of their masses is

(a) 1 : 2

(b) 1 : 1

(c) 4 : 1

(d) 1 : 4

22. A shell of mass m moving with a velocity v suddenly breaks into 2 pieces. The part having mass $m/4$ remains stationary. The velocity of other part will be

(a) v

(b) $2v$

(c) $\frac{3}{4}v$

(d) $\frac{4}{3}v$

23. A rod elongates by l when a body of mass M is suspended from it. The work done is
 (a) Mgl (b) $(\frac{1}{2})Mgl$
 (c) $2Mgl$ (d) Zero
24. A ball of mass m collides with another ball of same mass at rest, with velocity v . If coefficient of restitution is e , then the ratio of the velocities of first and second ball after the collision is
 i) $(1-e)/(1+e)$
 ii) $(1+e)/(1-e)$
 iii) $(1-e)/2$
 iv) $(1+e)/2$
25. A body loses half its velocity on penetrating 3 cm in a wooden block. How much will it penetrate more before coming to rest?
 (a) 1 cm (b) 4cm
 (c) 2 cm (d) 3 cm
26. The work done by the net force during the displacement of a 1.0 kg body from $x=0$ to $x=2$ m in a straight line with velocity $v = ax^{3/2}$ where $a = 5 \text{ m}^{-1/2} \text{ s}^{-1}$ is
 i) 100 J
 ii) 25 J
 iii) 15 J
 iv) 125 J
27. A body skids and stop in 20s second after moving 10 m. The force on a body due to road is 500 N and is directly opposed to the motion. The work done by the body on the road is
 i) + 500 J
 ii) - 1000 J
 iii) Zero
 iv) + 15000 J
28. A ball moving with a speed of 20m/sec strikes an identical ball at rest, such that after collision, the direction of each ball makes an angle of 30 degree with horizontal line of motion. Find the sum of the speeds of the two balls after collision ?
 i) $4/7 \text{ m/s}$
 ii) $20/\sqrt{3} \text{ m/s}$
 iii) $40/\sqrt{3} \text{ m/s}$
 iv) $\sqrt{3}/7 \text{ m/s}$
29. Two masses one is 4.5 times heavier than the other are dropped from same height. The relation between their momentum p_1 and p_2 , just before they hit the ground is
 i) $p_2 = p_1$
 ii) $p_2 = 4.5 p_1$
 iii) $p_2 = 9.0 p_1$
 iv) $p_1 = 9.0 p_2$
30. A spring of force constant k is cut into two pieces of lengths x and y . The force constant of first part is
 i) $k_1 = k(1 + x/y)$
 ii) $k_1 = k(1 + y/x)$

- iii) $k_1 = k(1 - x/y)$
 iv) $k_1 = k(x/y - 1)$

ANSWERS(MULTIPLE CHOICE QUESTIONS)

1. a 2. b 3. a 4. c 5. d 6. b 7. c 8. c 9. d 10. b 11. c 12. b 13. a
 14. d 15. c 16. d 17. d 18. b 19. a 20. d 21. d 22. d 23. b 24.(i) 25. A 26.(i)
 27. (iii) 28. (iii) 29. (ii) 30. (ii)

MULTIPLE CHOICE QUESTIONS(more than one correct answer)

1. One end of a light spring of spring constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement, the work done by the spring is $\frac{1}{2}kx^2$. The possible cases are

- (a) the spring was initially compressed by a distance x and was finally in its natural length
 (b) it was initially in its natural length and finally in a compressed position
 (c) it was initially stretched by a distance x and finally was in its natural length
 (d) it was initially in its natural length and finally in a stretched position.

2. The kinetic energy of a particle continuously increases with time

- (a) The resultant force on the particle must be parallel to the velocity at all instants
 (b) The resultant force on the particle must be at an angle less than 90° all the time
 (c) The magnitude of its linear momentum is increasing continuously
 (d) Its height above the ground level must continuously decrease.

3. You lift a suitcase from the floor and keep it on a table. The work done by you on the suitcase does not depend on

- (a) the weight of the suit case
 (b) the path taken by the suitcase
 (c) the time taken by you in doing so
 (d) your weight

4. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that

- (a) it moves in a circular path (b) its velocity is constant
 (c) its acceleration is constant (d) its kinetic energy is constant.

5. In head on elastic collision of two bodies of equal masses

- (a) the speeds are interchanged (b) the velocities are interchanged
 (c) the faster body slows down and the slower body speeds up (d) the momenta are interchanged

ANSWERS, MULTIPLE CHOICE QUESTIONS(more than one correct answer)

1. a, c 2. b, c 3. b, c, d 4. a, d 5. all

UNIT-V - MOTION OF SYSTEM OF PARTICLES

FORMULAE:-

1. Position vector of CM
$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

2. Velocity of CM
$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

3. Acceleration of CM
$$\vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i}$$

4. Equation of motion of CM
$$M\vec{a}_{cm} = M \frac{d\vec{v}_{cm}}{dt} = M \frac{d^2\vec{r}_{cm}}{dt^2}$$

5. If origin is at CM then
$$\sum m_i \vec{r}_i = \vec{0}$$

6. Position vector of CM of continuous mass distribution
$$\vec{r}_{cm} = \frac{\int \vec{r} dm}{\int dm}$$

7. Position of CM of different bodies:-

Body	Position of CM
Uniform thin rod	Centre of Rod
Uniform hollow/solid sphere	Centre of sphere
Uniform Circular Ring/Disc	Centre of Ring/Disc
A plane lamina (Rectangular, Square, Parallelogram)	Point of intersection of diagonals
Triangular Plane lamina	Point of intersection of medians
Cubical block	Point of intersection of diagonals
Hollow/Solid cylinder	Middle point of the axis of cylinder
Solid cone	On the axis of cone at a height $\frac{3h}{4}$ from vertex, where h is height
Hollow cone	On the axis of cone at a height $\frac{h}{3}$ from vertex, where h is height
Hollow hemisphere	On the axis of symmetry at a height $\frac{R}{2}$ from centre, where R is radius

Solid hemisphere

On the axis of symmetry at a height $\frac{3R}{8}$
from centre, where R is radius

8. Torque $\vec{\tau} = \vec{r} \times \vec{F}$

9. Angular Momentum $\vec{L} = \vec{r} \times \vec{p}$

where \vec{r} is position vector & \vec{p} is linear momentum

10. MI of a system of particles $I = \sum m_i r_i^2$,

where m = mass of particle & r = perpendicular distance of the particle from the axis of rotation

11. MI of a rigid body / continuous mass distribution

$$I = \int r^2 dm$$

12. Theorem of perpendicular axes $I_z = I_x + I_y$

13. Theorem of parallel axes $I = I_{cm} + Md^2$

14. MI of some regular bodies

Body	Axis of Rotation	Moment of Inertia
Uniform thin circular ring	Passing through CM & perpendicular to plane	MR^2
	Diameter	$\frac{1}{2}MR^2$
	Tangent in the plane	$\frac{3}{2}MR^2$
Uniform thin circular disc	Tangent perpendicular to Plane	$2MR^2$
	Passing through CM & perpendicular to plane	$\frac{1}{2}MR^2$
	Diameter	$\frac{1}{4}MR^2$
	Tangent in the plane	$\frac{5}{4}MR^2$
	Tangent perpendicular to Plane	$\frac{3}{2}MR^2$

Annular disc of inner radius R_1 and outer radius R_2	Passing through CM & perpendicular to plane	$\frac{1}{2}M(R_1^2 + R_2^2)$
Solid Cylinder	Geometrical Axis	$\frac{1}{2}MR^2$
	Passing through centre & perpendicular to length	$M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$
Hollow Cylinder	Geometrical Axis	MR^2
	Passing through centre & perpendicular to length	$M\left[\frac{L^2}{12} + \frac{R^2}{2}\right]$
Hollow sphere	Diameter	$\frac{2}{3}MR^2$
Solid Sphere	Diameter	$\frac{2}{5}MR^2$
Thin Uniform Rod	Passing through centre & perpendicular to length	$\frac{ML^2}{12}$
	Passing through edge & perpendicular to length	$\frac{ML^2}{3}$
Rectangular Lamina	Passing through CM & perpendicular to plane	$\frac{M}{12}[l^2 + b^2]$
Elliptical Disc	Passing through CM & perpendicular to plane	$\frac{M}{4}[a^2 + b^2]$
Solid Cone	Joining vertex and centre of base	$\frac{3}{10}MR^2$

15. Radius of Gyration $I = MK^2$

For a system of identical particles $K = \sqrt{\frac{r_1^2 + r_2^2 + \dots + r_n^2}{n}}$

16. Equations of rotational motion

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

17. Comparison of Translational and Rotational Motion

Translational Motion

Rotational Motion

Mass (m)

Moment of Inertia (I)

Linear Displacement (s)

Angular Displacement (θ)

Linear Velocity (v)

Angular Velocity (ω)

Linear Acceleration (a)

Angular Acceleration (α)

Linear momentum ($p = mv$)

Angular momentum ($L = I\omega$)

Force ($F = ma = \frac{dp}{dt}$)

Torque ($\tau = I\alpha = \frac{dL}{dt}$)

Kinetic energy ($K_T = \frac{1}{2}mv^2 = \frac{p^2}{2m}$)

Rotational KE ($K_R = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$)

Work ($W = Fs$)

Work ($W = \tau\theta$)

Power ($P = Fv$)

Power ($P = \tau\omega$)

18. Rolling on an Inclined plane

Velocity at lowest point $v = \sqrt{\frac{2gh}{1 + \frac{K^2}{R^2}}}$

Acceleration in motion $a = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}}$

Time of descent $t = \frac{1}{\sin \theta} \sqrt{\frac{2h(1 + \frac{K^2}{R^2})}{g}}$

19. Motion of a point mass connected to a string which is wound around a solid body (Cylinder, Disc)

Downward acceleration $a = \frac{g}{1 + \frac{I}{mR^2}}$

Tension in string $T = mg \left[\frac{I}{I + mR^2} \right]$

Velocity of point mass $v = \sqrt{\frac{2gh}{1 + \frac{I}{mR^2}}}$

Angular velocity of rigid body

$$w = \sqrt{\frac{2mgh}{I + mR^2}}$$

SHORT -CUT FORMULAE:-

1. If some area is removed from a rigid body, the position of centre of mass of the remaining portion is given by:-

$$r_{cm}^{\rightarrow} = \frac{m_1 r_1^{\rightarrow} - m_2 r_2^{\rightarrow}}{m_1 - m_2} = \frac{A_1 r_1^{\rightarrow} - A_2 r_2^{\rightarrow}}{A_1 - A_2}$$

2. If two masses are connected by a light inextensible string passing over a frictionless pulley, the acceleration of their centre of mass is given by:-

$$a_{cm} = \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2} g$$

3. Moment of inertia of two point masses about their centre of mass

$$I = \left[\frac{m_1 m_2}{m_1 + m_2} \right] r^2 = \mu r^2, \text{ where } \mu = \text{reduced mass of the system}$$

4. Time period of a compound pendulum

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$\text{where } L = \frac{l^2 + k^2}{l}$$

l = distance of CM from point of suspension

k = radius of gyration about parallel axis passing through CM

MULTIPLE CHOICE QUESTION WITH 1 ANSWER CORRECT

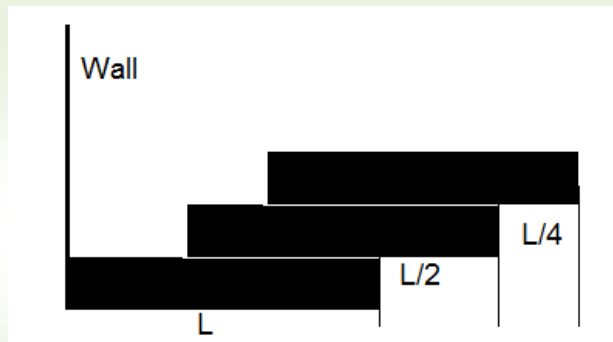
1. A system consists 3 particles each of mass m located at points (1,1), (2,2) and (3,3). The co-ordinates of the centre of mass are:-

- (a) (6,6)
- (b) (3,3)
- (c) (1,1)
- (d) (2,2)

2. Two spherical bodies of mass M and 5M and radii R and 2R respectively are released in free space with initial separation between their centres equal to 12R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is -

- (a) 1.5 R
- (b) 2.5 R
- (c) 4.5 R
- (d) 7.5 R

3. Three bricks each of length L and mass M are arranged as shown from the wall. The distance of the centre of mass of the system from the wall is-



- a) $L/4$
- (b) $L/2$
- (c) $3L/2$
- (d) $11L/12$

4. Three masses are placed on the x -axis, 300g at origin, 500g at $x=40\text{cm}$ and 400g at $x=70\text{cm}$. The distance of the centre of mass from origin is -

- (a) 40 cm
- (b) 45 cm
- (c) 50 cm
- (d) 30 cm

5. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω . Its centre of mass rises to a maximum height of -

- (a) $\frac{1}{3} \frac{l^2 \omega^2}{g}$
- (b) $\frac{1}{6} \frac{l\omega}{g}$

(c) $\frac{1}{2} \frac{l^2 \omega^2}{g}$

(d) $\frac{1}{6} \frac{l^2 \omega^2}{g}$

6. If linear density of a rod of length 3m varies as $\lambda=2+x$, then the position of the centre of mass of the rod is-

(a) $\frac{7}{3}$ m

(b) $\frac{12}{7}$ m

(c) $\frac{10}{7}$ m

(d) $\frac{9}{7}$ m

7. Centre of mass of 3 particles of mass 10kg, 20kg and 30kg is at (0,0,0). Where should a particle of mass 40kg be placed so that the centre of mass of the system will be at (3,3,3)?

(a) (0,0,0)

(b) (7.5,7.5,7.5)

(c) (1,2,3)

(d) (4,4,4)

8. n - particles of masse m each are placed on the same line at distances l, 2l, 3l, -----nl cm from a fixed point on the line. The distance of the centre of mass of the system from the fixed point would be-

(a) $\frac{(2n+1)l}{3}$

(b) $\frac{nl}{2}$

(c) $\frac{(n+1)l}{2}$

(d) $\frac{n(n+1)l}{2}$

9. If the earth shrinks suddenly such that its mass does not change but radius decreases to one quarter of its original value, then the duration of a day would be-

(a) 96 hrs

- (b) 48 hrs
- (c) 6 hrs
- (d) 1.5 hrs

10. A wheel has a speed of 1200 revolutions per minute and is made to slow down at a rate of 4 radian/s^2 . The number of revolutions it makes before coming to rest is-

- (a) 157
- (b) 314
- (c) 628
- (d) 722

11. A mass m hangs with the help of a light string wrapped around a pulley on a frictionless bearing. Assuming the pulley to be a uniform circular disc of mass m and radius R , the acceleration of the mass m would be-

- (a) $\frac{3}{2}g$
- (b) $\frac{2}{3}g$
- (c) g
- (d) $\frac{1}{3}g$

12. Two moment of inertia of a semicircular ring of mass M and radius R about an axis passing through centre and perpendicular to the plane of the ring is -

- (a) MR^2
- (b) $\frac{1}{2}MR^2$
- (c) $\frac{1}{4}MR^2$
- (d) $2MR^2$

513. Four thin rods each of mass M and length L form a square. The moment of inertia of the system about an axis passing through the centre and perpendicular to the plane of the square would be-

- (a) $\frac{4}{3}ML^2$

(b) $\frac{1}{3} ML^2$

(c) $\frac{1}{6} ML^2$

(d) $\frac{2}{3} ML^2$

14. Four point masses each of value m are placed at the corners of a square ABCD of side L . The moment of inertia of the system about an axis passing through A and parallel to BD would be-

(a) $\sqrt{3} mL^2$

(b) $3 mL^2$

(c) mL^2

(d) $2 mL^2$

15. Four particles each of mass m are placed at the corners of a square of side L . the radius of gyration of the system about an axis passing through centre and perpendicular to the plane of the square would be-

(a) $\frac{1}{\sqrt{2}} L$

(b) $\frac{1}{2} L$

(c) L

(d) $\sqrt{2} L$

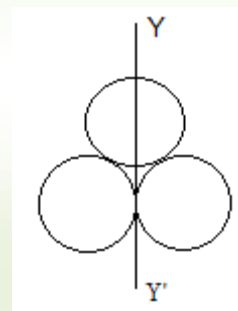
16. Three rings each of mass M and radius R are arranged as shown in the figure. The moment of inertia of the system about axis YY' would be-

(a) $3MR^2$

(b) $\frac{3}{2} MR^2$

(c) $5MR^2$

(d) $\frac{7}{2} MR^2$



17. The moment of inertia of a square plate of mass m and side L about an axis passing through one of corners and perpendicular to the plane of the plate is-

(a) $\frac{1}{12} ML^2$

(b) $\frac{7}{12} ML^2$

(c) $\frac{5}{6} ML^2$

(d) $\frac{2}{3} ML^2$

18. A rod of length L and mass M is bent to form a semi - circular ring. The moment of inertia of the semi - circular ring about its diameter would be-

(a) $\frac{ML^2}{2\pi^2}$

(b) $\frac{ML^2}{\pi^2}$

(c) $\frac{ML^2}{4\pi^2}$

(d) $\frac{2ML^2}{\pi^2}$

19. The moment of inertia of a thin uniform rod about a perpendicular axis passing through one end is I_1 . the same rod is bent into a ring and its moment of inertia about a diameter is I_2 . The ratio I_1/I_2 would be-

(a) $\frac{\pi^2}{3}$

(b) $\frac{2\pi^2}{3}$

(c) $\frac{4\pi^2}{3}$

(d) $\frac{8\pi^2}{3}$

20. From a uniform circular disc of radius R and mass 9M, a small disc of mass M and radius R/3 is removed concentrically. The moment of inertia of the remaining disc about an axis passing through centre and perpendicular to the plane of the disc would be-

(a) $\frac{40}{9} MR^2$

(b) $\frac{4}{9}MR^2$

(c) $4MR^2$

(d) MR^2

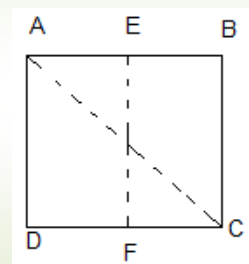
21. A rod of length L composed of a uniform length $L/2$ of wood whose mass is m_w and a uniform length $L/2$ of brass whose mass is m_b . The moment of inertia of the rod about an axis perpendicular to the length and passing through centre will be-

(a) $\frac{1}{6}(m_w + m_b)L^2$

(b) $\frac{1}{3}(m_w + m_b)L^2$

(c) $\frac{1}{12}(m_w + m_b)L^2$

(d) $\frac{1}{2}(m_w + m_b)L^2$



22. For the given uniform square lamina ABCD with centre O -

(a) $\sqrt{2} I_{AC} = I_{EF}$

(b) $I_{AC} = 3 I_{EF}$

(c) $I_{AC} = I_{EF}$

(d) $I_{AC} = \sqrt{2} I_{EF}$

23. From a disc of radius R and mass M , a circular hole of diameter R , whose rim passes through the centre is cut. What is the moment of inertia of the remaining part about a perpendicular axis passing through the centre?

(a) $\frac{15MR^2}{32}$

(b) $\frac{13MR^2}{32}$

(c) $\frac{11MR^2}{32}$

(d) $\frac{9MR^2}{32}$

24. Let \vec{F} be the force acting on a particle having position vector \vec{r} and $\vec{\tau}$ be the torque of this force about origin. Then-

(a) $\vec{r} \cdot \vec{\tau} = 0$ and $\vec{F} \cdot \vec{\tau} = 0$

(b) $\vec{r} \cdot \vec{\tau} = 0$ and $\vec{F} \cdot \vec{\tau} \neq 0$

(c) $\vec{r} \cdot \vec{\tau} \neq 0$ and $\vec{F} \cdot \vec{\tau} = 0$

(d) $\vec{r} \cdot \vec{\tau} \neq 0$ and $\vec{F} \cdot \vec{\tau} \neq 0$

25. A rod of weight W is supported by two parallel knife edges A and B, and is in equilibrium in horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at a distance x from A. The normal reaction on A is-

(a) $\frac{Wd}{x}$

(b) $\frac{W(d-x)}{x}$

(c) $\frac{W(d-x)}{d}$

(d) $\frac{Wx}{d}$

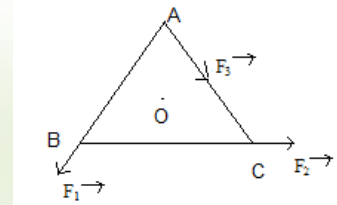
26. ABC is an equilateral triangle with centre O. $F_1 \rightarrow$, $F_2 \rightarrow$ and $F_3 \rightarrow$ represent three forces acting on the triangle along the sides AB, BC and AC respectively. If total torque about O is zero then -

(a) $F_3 = F_1 + F_2$

(b) $F_3 = F_1 - F_2$

(c) $F_3 = 2(F_1 + F_2)$

(d) $F_3 = (F_1 + F_2)/2$



27. A thin circular disc of mass M and radius R is rotating about its axis with constant angular velocity ω . Four particles each of mass m are kept gently to the opposite ends of two perpendicular diameters of the disc. The angular velocity of the disc would be -

(a) $\frac{M\omega}{M + 4m}$

(b) $\frac{\omega(M + 4m)}{M}$

(c) $\frac{M\omega}{(M + 8m)}$

(d) $\frac{(M - 4m)\omega}{(M + 8m)}$

28. A particle moves with a constant velocity v in a straight line parallel to X-axis. The angular momentum of the particle with respect to origin is -

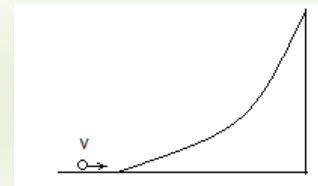
- (a) Zero
- (b) Constant
- (c) Continuously increases
- (d) Continuously decreases

29. A solid spherical ball rolls on an inclined plane without slipping. The ratio of its rotational and total kinetic energy is -

- (a) $\frac{2}{5}$
- (b) $\frac{2}{7}$
- (c) $\frac{3}{5}$
- (d) $\frac{3}{7}$

30. A small object of uniform density rolls without slipping up a curved surface with an initial velocity v . It reaches up to a maximum height of $3v^2/4g$ with respect to the initial position. The object is -

- (a) Ring
- (b) Solid sphere
- (c) Hollow sphere
- (d) Disc



MULTIPLE CHOICE QUESTION WITH MORE THAN 1 ANSWER CORRECT

1. A solid cylinder of mass m and radius r is rolling on a rough inclined plane of inclination θ . The coefficient of friction between the cylinder and the incline is μ . Then -

- (a) Frictional force is always $\mu mg \cos\theta$
- (b) Friction is dissipative force
- (c) By decreasing θ , frictional force decreases
- (d) Friction opposes translation and supports rotation

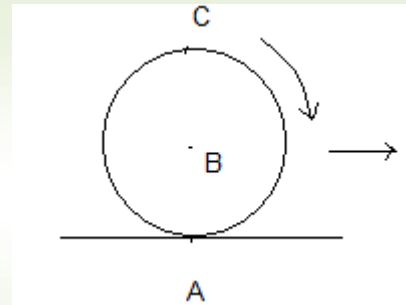
2. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is point of contact, B is the centre and C is the top most point of the sphere. Then -

(a) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$

(b) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(c) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(d) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$



3. For a uniform circular disc of mass M and radius R, the moment of inertia about :-

(a) A diameter is $\frac{MR^2}{4}$

(b) A tangent perpendicular to the plane is $\frac{5MR^2}{4}$

(c) An axis in the plane of disc at a distance R/2 from centre is $\frac{MR^2}{2}$

(d) The geometrical axis is $\frac{MR^2}{2}$

4. The radius of the earth suddenly decreases to half the present value keeping mass constant, then

(a) Angular speed will decrease

(b) Duration of a day will be 8 hrs

(c) Angular momentum will be conserved

(d) Rotational kinetic energy of the earth will increase

5. Two particles of masses 10g and 20g are placed 30cm apart. The centre of mass of the system is:-

(a) Midway between the particles

(b) 20cm from particle of 10g

(c) 10cm from particle of 20g

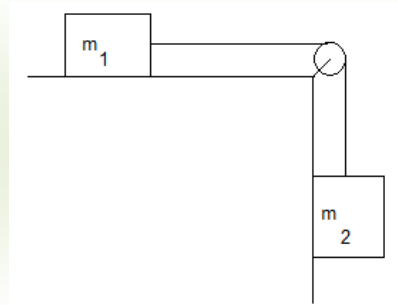
(d) 10cm from particle of 10g

6. A uniform circular ring of mass 20g and radius 2cm is rotating about geometrical axis with initial angular speed of 2 rad/sec. It is subjected to a constant angular acceleration of 3 rad/sec² for 2 seconds. Then -

- (a) Angular velocity after 2 second will be 8 rad/sec
- (b) Moment of inertia about geometrical axis is 40 g-cm²
- (c) Rotational kinetic energy after 2 second will be 2560 erg
- (d) Initial angular momentum is 160 g-cm²/sec

7. All surfaces shown in the figure are smooth. System is released from rest. x and y components of the acceleration of centre of mass are given by:-

- (a) $(a_{cm})_x = \frac{m_1 m_2 g}{m_1 + m_2}$
- (b) $(a_{cm})_x = \frac{m_1 m_2 g}{(m_1 + m_2)^2}$
- (c) $(a_{cm})_y = \left(\frac{m_2}{m_1 + m_2}\right)^2 g$
- (d) $(a_{cm})_x = \left(\frac{m_2}{m_1 + m_2}\right) g$



8. The instantaneous angular position of a particle on a rotating wheel is given by $\theta(t) = 2t^3 - 6t^2 + 2$. Then-

- (a) Initial angular velocity is 2 rad/sec
- (b) Angular acceleration increases linearly with time
- (c) Angular acceleration decreases linearly with time
- (d) Torque will be zero at $t = 1$ sec

9. A particle with position vector \vec{r} has linear momentum \vec{p} . Then

- (a) Angular momentum acts along \vec{r}
- (b) Angular momentum acts perpendicular to \vec{r}
- (c) Angular momentum acts perpendicular to \vec{p}
- (d) Angular momentum acts perpendicular to \vec{r}

10. Two uniform circular discs of same mass and radii r and $2r$ roll from the top of inclined plane without slipping. Then -

- (a) Smaller disc has lesser speed at the bottom of inclined plane
- (b) Larger disc has lesser speed at the bottom of inclined plane

(c) Smaller disc reaches earlier at the bottom of inclined plane

(d) Larger disc reaches earlier at the bottom of inclined plane

ANSWER MCQ WITH 1 CORRECT ANSWER

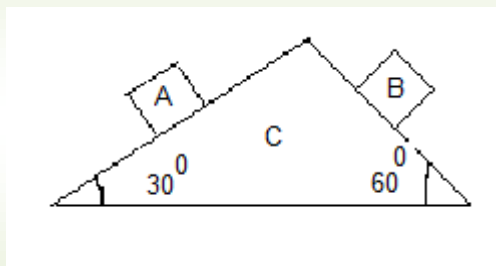
Q.No.	ANS	Q.No.	ANS	Q.No.	ANS	Q.No.	ANS	Q.No.	ANS
1	D	7	B	13	A	19	D	25	C
2	D	8	C	14	B	20	A	26	A
3	D	9	D	15	A	21	C	27	C
4	A	10	B	16	D	22	C	28	B
5	D	11	B	17	D	23	B	29	B
6	B	12	A	18	A	24	A	30	D

ANSWER MCQ WITH MORE THAN 1 CORRECT ANSWER

Q.No.	ANS	Q.No.	ANS	Q.No.	ANS	Q.No.	ANS	Q.No.	ANS
1	C, D	2	B,C	3	A,C,D	4	C,D	5	B,C
6	A,C,D	7	B,C	8	B,D	9	B,C	10	A,D

SUBJECTIVE QUESTIONS

Q1. Two blocks A and B of equal masses are released on the two sides of a fixed wedge C as shown in the figure. Find the acceleration of their centre of mass. Neglect friction.



Q2. A square lamina of side a and a circular lamina of diameter a are placed in contact as shown in the figure. Find the distance of the centre of mass of the system from centre of the square O if the mass per unit area is same for both of the lamina.

3. A rigid sphere of uniform mass density ρ and radius R has two smaller spheres of radii $R/2$ hollowed out of it as shown in the figure. Find the moment of inertia of remaining body about Y-axis.

4. A rod OA is hinged at its end O to the edge of a table. Initially the rod is kept horizontal and released from rest. Determine the acceleration of the centre of mass of the rod.

5. A rod of mass M and length L is suspended by a frictionless hinge at point O as shown in the figure. A point bullet of mass m moving with velocity v in horizontal direction strikes the end of the rod and gets embedded in it. Find the angular velocity acquired by the rod just after the collision.

SOLUTION TO SUBJECTIVE QUESTIONS

Q1. Equation of motion of two particles

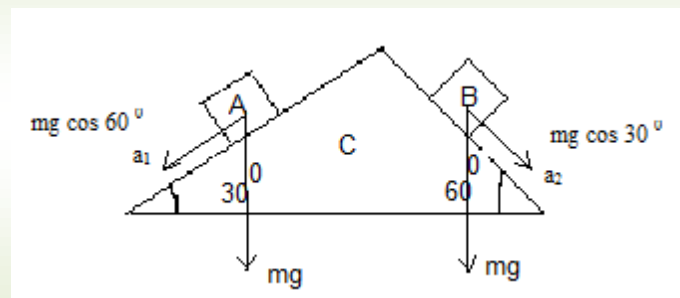
$$ma_1 = mg \cos 60^\circ$$

$$\Rightarrow a_1 = \frac{1}{2}g$$

and

$$ma_2 = mg \cos 30^\circ$$

$$\Rightarrow a_2 = \frac{\sqrt{3}}{2}g$$



Acceleration of centre of mass

$$\vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i} = \frac{ma_1\vec{a}_1 + ma_2\vec{a}_2}{m+m} = \frac{1}{2}(a_1\vec{a}_1 + a_2\vec{a}_2)$$

$$\Rightarrow a_{cm} = \frac{1}{2}\sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos 90^\circ} = \frac{1}{2}\sqrt{\frac{g^2}{4} + \frac{3g^2}{4}} = \frac{g}{2}$$

Q2.

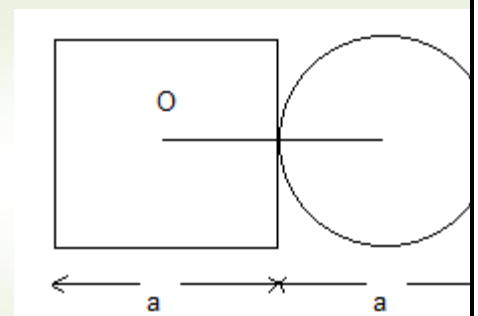
Let mass per unit area of each lamina be m . Then

the mass of square lamina

$$m_1 = m a^2$$

and the mass of circular lamina

$$m_2 = m \pi a^2/4$$



Centre of mass of the system

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$\Rightarrow x_{cm} = \frac{ma^2 X 0 + m\pi \frac{a^2}{4} X a}{ma^2 + m\pi \frac{a^2}{4}}$$

$$\Rightarrow x_{cm} = \frac{\pi a}{4 + \pi}$$

Q3.

Mass of each hollowed sphere

$$m = \frac{4}{3} \pi \left(\frac{R}{2}\right)^3 \rho = \frac{1}{6} \pi R^3 \rho$$

MI of each hollowed sphere about Y -axis

$$I_1 = \frac{2}{5} m \left(\frac{R}{2}\right)^2 + m \left(\frac{R}{2}\right)^2 = \frac{7}{5} \left(\frac{1}{6} \pi R^3 \rho\right) \left(\frac{R^2}{4}\right) = \frac{7}{120} \pi R^5 \rho$$

MI of rigid sphere about Y-axis

$$I_2 = \frac{2}{5} \left(\frac{4}{3} \pi R^3 \rho\right) R^2 = \frac{8}{15} \pi R^5 \rho$$

MI of the remaining body

$$I = I_2 - 2I_1 = \frac{8}{15} \pi R^5 \rho - 2 \times \frac{7}{120} \pi R^5 \rho = \frac{5}{12} \pi R^5 \rho$$

Q4.

Let m be the mass and L be the length of the rod.

Equation of rotational motion of rod when released

from rest in horizontal position

$$\tau = I\alpha \Rightarrow mgX \frac{L}{2} = \frac{1}{3} mL^2 \alpha \Rightarrow \alpha = \frac{3g}{2L}$$

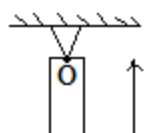
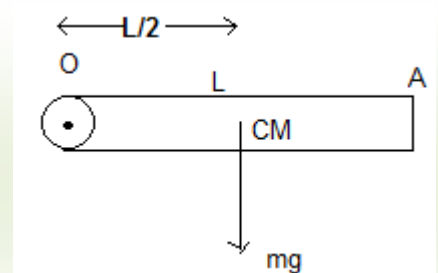
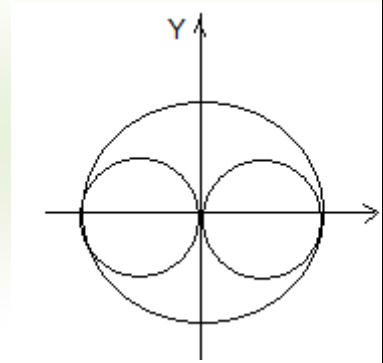
Hence acceleration of CM of rod

$$a = r\alpha = \frac{L}{2} X \frac{3g}{2L} = \frac{3g}{4}$$

Q5.

Applying the law of conservation of angular

momentum about hinge O,



$$mvL = (mL^2 + \frac{1}{3}ML^2)\omega$$

$$\Rightarrow \omega = \frac{3mvL}{(3mL^2 + ML^2)}$$

$$\Rightarrow \omega = \frac{3mv}{(3m + M)L}$$

Unit-VI -GRAVITATION

Formulae & Important Concepts

*Gravitational force is attractive force between two masses m_1 and m_2 separated by a distance and it is given by $F = -G (M_1M_2/r^2)$, where G is universal gravitational constant, $G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$. The negative sign shows that force is attractive.

★ Dimensional formula of G is $[M^{-1}L^3T^{-2}]$.

★ The gravitation is the central force. It acts along the line joining the particles. It is a conservative force. The work-done by it is independent of the path followed. This force is attractive in nature.

★ It is the weakest force in nature. It is 10^{36} times smaller than electric force.

★ Gravitation is independent of the presence of other bodies around it.

★ The gravitational pull of the earth is called gravity.

★ The gravitation forces between two bodies are action-reaction pairs. The law of gravitation is called universal as it is applicable to all bodies, irrespective of their size, shape or position. This force does not depend upon the state of the bodies, nature of the intervening medium, temperature and other physical condition of the bodies.

★ In motion of the planets and satellites, force of gravitation provides the necessary centripetal force due to which earth revolves around the sun and moon around the earth.

★ The value of acceleration due to gravity increases as we go from equator to the pole.

★ The value of the acceleration due to gravity on the moon is about one sixth of that on the earth and on the sun is about 27 times that on the earth.

★ Among the planets, the g is minimum on the mercury.

★ Acceleration due to gravity on the surface of the earth is given by, $g = \frac{GM}{R^2}$, where M is the mass of the earth and R is the radius of the earth.

★ Acceleration due to gravity at a height h above the surface of the earth is given by

$$g_h = \frac{GM}{(R+h)^2} \approx g(1 - 2h/R)$$

The approximation is true when $h \ll R$.

★ Value of g at depth d from earth's surface

$$(a) g' = g \left[1 - \frac{d}{R} \right] \quad (b) g' = \frac{GM}{R^3} (R-d)$$

★ The value of g at height h from the surface and $h \ll R$ is $g' = g \left(1 - \frac{2h}{R} \right)$.

★ The value of g at latitude λ is

(a) $g' = g - \omega^2 R_e \cos^2 \lambda$

(b) At the equator $\lambda = 0$. $\therefore g' = g - \omega^2 R_e$

(c) At the poles $\lambda = \pi/2$. $\therefore g' = g$.

★ The decrease in g with latitude is caused by the rotation of the earth about its own axis. A part of g is used to provide the centripetal acceleration for rotation about the axis.

★ The rotation of the earth about the sun has no effect on the value of g .

★ Decrease in g in going from poles to the equator is about 0.35%.

★ The weight of the body varies in the same manner as the g does. ($W = mg$).

★ The gravitational force to attraction acting on a body of unit mass at any point in the gravitational field is defined as the intensity of gravitational field E_g at that point.

$$E_g = \frac{F}{m} = \frac{GM}{r^2}$$

★ The gravitational potential energy of a mass m at a point above the surface of the earth at a height h is given by $\frac{-GMm}{R+h}$. The negative sign implies that as R increases, the gravitational potential energy decreases and becomes zero at infinity.

★ The body is moved from the surface of earth to a point at a height h above the surface of earth then change in potential energy will be mgh .

★ Gravitational potential at a point above the surface of the earth at a height h is $-\frac{GM}{(R+h)}$. Its unit is joule/kilogram.

★ Gravitational mass, M_g is defined by Newton's law of gravitation.

$$M_g = \frac{F_g}{g} = \frac{W}{g} = \frac{\text{Weight of body}}{\text{Acceleration due to gravity}}$$

$$\frac{(M_1)g}{(M_2)g} = \frac{F_{g1}g_2}{g_1F_{g2}}$$

★ Let ω_0 = angular speed of the satellite, v_0 = orbital speed of the satellite, then $v_0 = (R+h)\omega_0$, where R = radius of the earth and h = height of the satellite above the surface of the earth. Let g = acceleration due to gravity on the surface of the earth, T = time period of the satellite, M = mass of the earth. Then different quantities connected with satellite at height h are as follows :

(a) $\omega_0 = \left[\frac{GM}{(R+h)^3} \right]^{1/2} = R \left[\frac{g}{(R+h)^3} \right]^{1/2}$.

(b) $T = \frac{2\pi}{\omega_0}$ and frequency of revolution, $\nu = \frac{\omega_0}{2\pi}$.

$$(c) v_0 = \left[\frac{GM}{R+h} \right]^{1/2} \quad R = \left[\frac{g}{R+h} \right]^{1/2}$$

Very near the surface of the earth, we get the values by putting $h = 0$. That is :

$$(i) \omega_0 = \left[\frac{GM}{R^3} \right]^{1/2} = \left[\frac{g}{R} \right]^{1/2}$$

$$(ii) v_0 = \left[\frac{GM}{R} \right]^{1/2} = [gR]^{1/2}$$

$$(iii) T \approx 2\pi \left[\frac{R}{g} \right]^{1/2} = 5078 \text{ sec} = 1 \text{ hour } 24.6 \text{ minute.}$$

★ Altitude or height of satellite above the earth's surface, $h = \left(\frac{T^2 R^2 g}{4\pi^2} \right)^{1/3} - R$.

★ Angular momentum, $L = mv_{(R+h)} = \left[m^2 GM (R+h) \right]^{1/2}$.

★ Above the surface of the earth, the acceleration due to gravity varies inversely as the square of the distance from the centre of the earth.

$$g' = \frac{gR^2}{(R+h)^2}$$

★ The gravitational potential energy of a satellite of mass m is $U = \frac{-GMm}{r}$, where r is the radius of the orbit.

It is negative.

★ Kinetic energy of the satellite is $K = \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$.

★ Total energy of the satellite $E = U + K = -\frac{GMm}{2r}$

—ve sign indicates that it is the binding energy of the satellite.

★ Total energy of a satellite at a height equal to the radius of the earth is given by

$$-\frac{GMm}{2(R+R)} = -\frac{GMm}{4R} = \frac{1}{4}mgR$$

where $g = \frac{GM}{R^2}$ is the acceleration due to gravity on the surface of the earth.

When the total energy of the satellite becomes zero or greater than zero, the satellite escapes from the gravitational pull of the earth.

★ If the radius of planet decreases by $n\%$, keeping the mass unchanged, the acceleration due to gravity on its surface increases by $2n\%$ i.e. $\frac{\Delta g}{g} = \frac{2\Delta R}{R}$.

★ If the mass of the planet increases by $m\%$ keeping the radius constant, the acceleration due to gravity on its surface increases by $m\%$ $\left[\frac{\Delta g}{g} = \frac{\Delta M}{M} \right]$ where $R = \text{constant}$.

- ★ If the density of planet decreases by $p\%$ keeping the radius constant, the acceleration due to gravity decreases by $p\%$.
- ★ If the radius of the planet decreases by $q\%$ keeping the density constant, the acceleration due to gravity decreases by $q\%$.
- ★ For the planets orbiting around the sun, the angular speed, the linear speed and kinetic energy change with time but the angular momentum remains constant.
- ★ The minimum velocity with which a body must be projected in the atmosphere so as to enable it to just overcome the gravitational attraction of the earth is called escape velocity. i.e. $v_e = \sqrt{2gR}$, where R = radius of earth.
- ★ The relation between orbital velocity of satellite and escape velocity is $v_e = \sqrt{2} v_0$. i.e. if the orbital velocity of a satellite revolving close to the earth happens to increase to $\sqrt{2}$ times, the satellite would escape.
- ★ There is no atmosphere on the moon because the escape velocity on the moon is low.
- ★ If the orbital radius of the earth around the sun be one fourth of the present value, then the duration of the year will be one eighth of the present value.
- ★ Weightlessness is a situation in which the effective weight of the body becomes zero. Circumstances when this condition arises.
 - (i) When the body is taken at the centre of the earth.
 - (ii) When a body is lying in a freely falling lift, ($a = g$).
 - (iii) When the body is inside a space craft or satellite which is orbiting around the earth.
- ★ **Kepler's first law (law of orbit)** : Every planet revolves around the sun in an elliptical orbit. The sun is situated at one focus of the ellipse.
- ★ **Kepler's second law (law of area)**: The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time. i.e. the areal velocity of planet around the sun is constant.
- ★ **Kepler's third law (law of period)**: The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semi-major axis of its elliptical orbit, i.e. $T^2 \propto a^3$, where a = semi-major axis of the elliptical orbit of the planet around the sun.
- ★ **Shape of orbit of a satellite**: The shape of orbit of a satellite depends upon its velocity of projection v from earth.
 - (a) If $v < v_0$, the satellite falls back to earth following a spiral path.
 - (b) If $v = v_0$, the satellite revolves in circular path/orbit around earth.
 - (c) If $v > v_0$, but less than escape velocity v_e , i.e. $v_0 < v < v_e$, the satellite shall revolve around earth in elliptical orbit.

- (d) If $v = v_e$, the satellite will escape away following a parabolic path.
 (e) If $v > v_e$, the satellite will escape away following a hyperbolic path.

★ **Geo-stationary satellite:**

- (a) Time period = 24 hour

It is synchronous with earth.

(b) The angular velocity of satellite must be in the same direction as that of the earth. It thus revolves around earth from west to east. Its relative angular velocity with respect to earth is zero.

(c) The orbit of satellite should be circular.

(d) The orbit should be in equatorial plane of earth. It contains centre of earth as well as equator.

(e) It should be at 36000 km from the surface of earth. It is thus (36000 + 6400) km or 42400 km from the centre of earth.

Radius of parking orbit = 42400 km.

(f) A satellite revolving in stable orbit does not require any energy from an external source. The work done by the satellite in completing its orbit is zero.

(g) Acceleration due to gravity is used up in providing centripetal acceleration to the satellite. Hence effective g inside the satellite is zero.

(h) Its orbital velocity = 3.1 km/sec

Its angular velocity = $\frac{2\pi}{24}$ radian/hour.

★ **For a satellite orbiting near earth's surface:**

(a) Time period = 84 minute approximately

(b) Orbital velocity = 8 km/sec

(c) Angular speed $\omega = \frac{2\pi \text{ radian}}{84 \text{ min}} = 0.00125 \frac{\text{radian}}{\text{sec}}$.

★ **Inertial mass and gravitational mass:**

(a) Inertial mass = $\frac{\text{force}}{\text{acceleration}}$

(b) Gravitational mass = $\frac{\text{weight of body}}{\text{acceleration due to gravity}}$

(c) They are equal to each other in magnitude.

(d) Gravitational mass of a body is affected by the presence of other bodies near it. Inertial mass of a body remains unaffected by the presence of other bodies near it.

Multiple Choice Questions

(With one option correct)

1. If the radius of earth decreases by 10%, the mass remaining unchanged, what will happen to the acceleration due to gravity?

- (a) Decrease by 19%
- (b) Increases by 19%
- (c) Decreases by more than 19%
- (d) Increases by more than 19%

2. The gravitational field in a region is given by 13 N/kg then the magnitude of the gravitational force acting on a particle of mass 2 kg placed at the origin, will be :

- (a) zero
- (b) 12N
- (c) 26N
- (d) 75 N

3. Three particles, each of mass 10^{-2} kg are brought from infinity to the vertices of an equilateral triangle of side 0.1 m, the work done is:

- (a) 2×10^{-8} J
- (b) 2×10^{-11} J
- (c) 2×10^{-12} J
- (d) 2×10^{-13} J

4. The ratio of acceleration due to gravity at a depth ' h ' below the surface of earth and at a height ' h ' above the surface for $h \ll$ radius of earth:

- (a) is constant
- (b) increases linearly with h
- (c) increases parabolically with h
- (d) decreases

5. A satellite of mass m moves along an elliptical path around the earth. The areal velocity of the satellite is proportional to:

- (a) m
- (b) m^{-1}
- (c) m^0
- (d) $m^{1/2}$

1

6. The angular momentum (L) of earth revolving round the sun is proportional to r^n , where ' r ' is the orbital radius of the earth. The value of n is: (assume the orbit to be circular)

- (a) $1/2$
- (b) 1
- (c) $-1/2$
- (d) 2

7. If F_1 is the magnitude of the force exerted on earth by moon and F_2 is the magnitude of force exerted on moon by earth then:

- (a) $F_1 > F_2$
- (b) $F_1 = F_2$
- (c) $F_1 < F_2$
- (d) None of these

8. A particle is fired vertically upwards with a speed of $v = \sqrt{0.8}v_e$, if the radius of the earth is ' R ' then the maximum height attained by the particle will be:

- (a) R
- (b) $R/2$
- (c) $2R$
- (d) $4R$

9. A planet has radius and mass, both half of those of the earth. The value of ' g ' on that planet will be:

- (a) 4.9m/s^2
- (b) 9.8m/s^2

(c) 19.6m/s^2

(d) 13.8 m/s^2

10. If the radius of the earth reduces by 4%, density remaining the same, then the escape velocity will:

(a) increase by 4%

(b) increase by 2%

(c) decrease by 4%

(d) decrease by 2%

11. A body is projected vertically from the earth's surface with velocity equal to half of the escape velocity. Maximum height reached by the body is:

(a) $5R/3$

(b) $R/3$

(c) $2R/3$

(d) $R/2$

12. The time period of artificial satellite in a circular orbit of radius R is T . The radius of the orbit in which time period is $8T$ is:

(a) $2R$

(b) $3R$

(c) $4R$

(d) $5R$

13. A body is projected with escape velocity 11.2 km/s from earth's surface. If the body is projected in a direction 30° angle to the vertical, its escape velocity in this case will be:

(a) 11.2km/s

(b) $11.2(1/\sqrt{2})\text{km/s}$

(c) $11.2(\sqrt{3}/2)\text{km/s}$

(d) none of these

14. The time period of a simple pendulum at the centre of earth is:

(a) zero

(b) infinite

(c) both (a) & (b)

(d) none of these

15. If the distance between the two particles is increased by 2%, then the force of attraction between them will:

(a) decrease by 6%

(b) decrease by 4%

(c) increase by 4%

(d) increase by 6%

16. How the gravitational constant will change if a brass plate is introduced between two bodies?

(a) No change

(b) Decreases

(c) Increases

(d) No sufficient data

17. Six particles each of mass m are placed at the corners of a regular hexagon of edge length a . If a point mass m_0 is placed at the centre of the hexagon, then the net gravitational force on the point mass m_0 is :

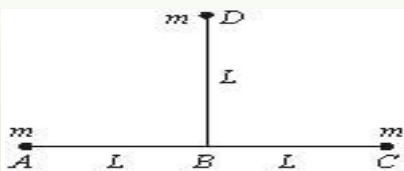
(a) $\frac{6Gm^2}{a^2}$

(b) $\frac{6Gmm_0}{a^2}$

(c) zero

(d) none of these

18. Two particles each of mass m are placed at A and C as such $AB = BC = L$. The gravitational force on the third particle placed at D at a distance L metre on the perpendicular bisector of the line AC , is:



(a) $\frac{Gm^2}{\sqrt{2}L^2}$ along BD

(b) $\frac{Gm^2}{\sqrt{2}L^2}$ along DB

(c) $\frac{Gm^2}{L^2}$ along AC

(d) none of these

19. In a hypothetical concept, electron of mass m_e revolves around nucleus due to gravitational force of attraction between electron and proton of mass m_p . If the radius of circular path of electron is r then the speed of electron is:

(a) $\sqrt{\frac{Gm_p m_e}{4r}}$

(b) $\sqrt{\frac{Gm_p m_e}{r}}$

(c) $\sqrt{\frac{Gm_p}{r}}$

(d) none of these

20. Suppose the gravitational force varies inversely as the n th power of the distance. then the time period of a planet in circular orbit of radius R around the sun will be proportional to :

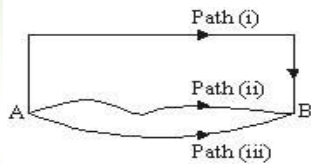
(a) R^n

(b) $R^{(n+1)/2}$

(c) $R^{(n-1)/2}$

(d) R^{-n}

21. A gravitational field is present in a region. A point mass is shifted from A to B , from different paths shown in the figure. If W_1 , W_2 and W_3 represent work done by gravitational force for respective paths, then:



- (a) $W_1 = W_2 = W_3$
- (b) $W_1 > W_2 > W_3$
- (c) $W_1 > W_3 > W_2$
- (d) None of these

22. Two identical spherical masses are kept at some distance as shown. Potential energy when a mass m is taken from surface of one sphere to the other:



- (a) increase continuously
- (b) decreases continuously
- (c) first increases then decreases
- (d) first decreases then increases

23. A point mass m_0 is placed at distance $R/3$ from the centre of a spherical shell of mass m and radius R . The gravitational force on the point mass m_0 is:

(a) $\frac{4Gmm_0}{R^2}$

(b) zero

(c) $\frac{9Gmm_0}{R^2}$

(d) none of these

24. n -particles each of mass m_0 are placed on different corners of a regular polygon of edge length a . The distance between vertex and centre of polygon is r_0 . The gravitational potential at the centre of the polygon is:

(a) $-\frac{Gnm_0}{r_0}$

(b) $-\frac{Gm_0}{r_0}$

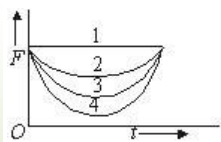
(c) $\frac{nGm_0}{r_0}$

(d) none of these

25. Suppose that in a region only gravitational field due to masses M_1 and M_2 are present. A particle of mass m goes from surface of M_1 to the surface of M_2 in a spaceship moving with constant velocity. Neglect all other objects besides M_1 , M_2 and m .



Which part of figure best represents the net gravitational force on the particle as a function of time?



(a) 1

(b) 2

(c) 3

(d) 4

26. A point mass of 10 kg is placed at the centre of earth. The weight of the point mass is:

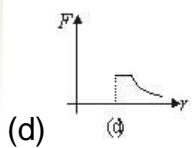
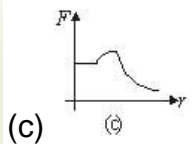
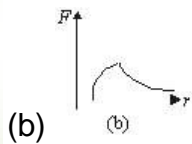
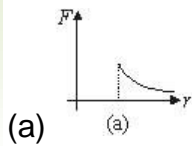
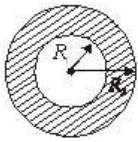
(a) zero

(b) 98N

(c)49N

(d) none of these

27. A sphere of mass M and radius R_2 has a concentric cavity of radius R_1 as shown in figure. The force F exerted by the sphere on a particle of mass m located a distance r from the centre of sphere varies as: ($0 \leq r \leq \infty$)



28. Three point masses each of mass m rotate in a circle of radius r with constant angular velocity ω due to their mutual gravitational attraction. If at any instant, the masses are on the vertex of an equilateral triangle of side a , then the value of ω is:

(a) $\sqrt{\frac{Gm}{a^3}}$

(b) $\sqrt{\frac{3Gm}{a^3}}$

(c) $\sqrt{\frac{Gm}{3a^3}}$

(d) None of these

29. A spherical mass of radius ($r=R/2$) is taken out from a uniform sphere of radius R and mass density ρ . The force which this sphere having a cavity will exert on a mass m placed at a distance of x from its centre ' x ' ($x < R$) is:

(a) $\frac{4\pi}{3} \rho G m R^3 \frac{1}{x^2}$

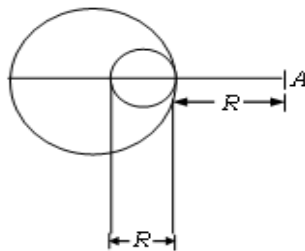
(b) $\frac{4\pi}{3} \rho G m R^3 \left[\frac{1}{x^2} - \frac{1}{2(2x-R)^2} \right]$

(c) $\frac{4Gm\rho}{3(R^2 + x^2)^{3/2}}$

(d) None of these

30.

A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at A, distant $2R$ from the centre of the sphere. A spherical cavity of radius $R/2$ is now made in the sphere as shown in the figure. The sphere with cavity now applies a gravitational force F_2 on the same particle placed at A. The ratio F_2/F_1 will be



(a) 1/4

(b) 3

(c) 7

(d) 7/9

Answer Key with Solutions

Question No.	Correct Option with Solution
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1	(d) Increases by more than 19%
2	<p>(c) 26 N</p> $F = \vec{E} m = 13 \times 2 = 26 \text{ N}$
3	<p>(d) $2 \times 10^{-13} \text{ J}$</p> <p>The work done in bringing each mass m from infinity to the vertices of equilateral triangle of side r is negative of the gravitational potential energy of the system.</p> $W = -U$ $= - \left(-3 \frac{Gm^2}{r} \right)$ $= \frac{3 \times 6.67 \times 10^{-11} \times 10^{-4}}{0.1} = 2 \times 10^{-13} \text{ J}$
4	<p>(b) increases linearly with h</p> <p>Let R be radius of earth and g the acceleration due to gravity on earth's surface. Then the desired ratio (say x) is:</p> $(h \ll R)$ $\approx 1 + \frac{h}{R}$ <p>From this expression we see that x increase linearly with h.</p>
5	<p>(c) m^0</p> <p>Areal velocity = constant, $L = mvr \sin \alpha$ (angular momentum) i.e. $\frac{dA}{dt} \propto m^0$</p>
6	(a) 1/2
105	

$$L = mvr \dots(i)$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \dots(ii) \quad \text{So } L = m(GMr)^{1/2} \text{ or } n = \frac{1}{2}$$

7

$$(b) F_1 = F_1$$

8

$$(d) 4R$$

$$h = \frac{R}{\left(\frac{ve^2}{v^2} - 1\right)}$$

9

$$1. (c) 19.6 \text{ m/s}^2$$

$$\text{so, } \frac{g_p}{g_e} = \frac{M_p}{M_e} \times \left(\frac{R/2}{R}\right)^2$$

$$= \left(\frac{M}{M/2}\right) \times \left(\frac{R/2}{R}\right)^2$$

$$= 2 \times \frac{1}{4}$$

$$= \frac{1}{2} = g_p = 2g_e = 2 \times 9.8 = 19.6 \text{ m/s}^2$$

10

(c) decrease by 4%

11

$$(b) \frac{R}{3}$$

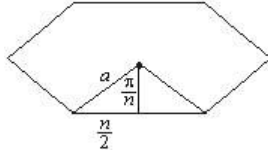
From conservation of mechanical energy

$$\frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{mgh}{\left(1 + \frac{h}{R}\right)} \quad \text{or} \quad \frac{1}{2}m\left(\frac{\sqrt{2gR}}{2}\right)^2 = \frac{mghR}{R+h} \quad \text{or} \quad h = \frac{R}{3}$$

12	<p>(c) $4R$</p> <p>$T^2 \propto R^3$</p> <p>$(8T)^2 \propto (R')^3$</p> <p>$\frac{T^2}{(8T)^2} = \left(\frac{R}{R'}\right)^3$ or $\left(\frac{1}{8}\right)^2 = \left(\frac{R}{R'}\right)^3$ or $R = 4R'$</p>
13	<p>(a) 11.2 km/s</p> <p>Escape velocity does not depend on angle of projection, so it will remain same i.e., 11.2 km/s.</p>
14	<p>(b) infinite</p> <p>$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{0}} = \infty$</p>
15	<p>(b) decrease by 4%</p> <p>$F = \frac{Gm_1m_2}{r^2}$ Or $F \propto \frac{1}{r^2}$</p> <p>$\frac{\Delta F}{F} = -2\frac{\Delta r}{r}$ Or $\frac{\Delta r}{r} = 2\%$ or $\frac{\Delta F}{F} = -2 \times 2\% = -4\%$ Negative sign shows decrease in force of attraction.</p>
16	<p>(a) No change</p>
17	<p>(c) zero</p> <p>If equal masses are placed at respective corners of a regular polygon, then the gravitational force on a point mass placed at centre of the regular hexagon is</p>

	zero.
18	(b) $\frac{Gm^2}{\sqrt{2}L^2}$ along DB
19	(c) $\sqrt{\frac{Gm_p}{r}}$ The force of attraction between electron and proton provides required centripetal force. $F = \frac{Gm_p m_e}{r^2} \text{ or } \frac{m_e v^2}{r} = \frac{Gm_p m_e}{r^2} \text{ or } v = \sqrt{\frac{Gm_p}{r}}$
20	(b) $R^{(n+1)/2}$
21	(a) $W_1 = W_2 = W_3$ The work done by gravitational force does not depend upon path. So, work done in all paths is same.
22	(c) first increases then decreases Centre point is unstable equilibrium position where potential energy is maximum.
23	(b) zero The gravitational field inside the spherical shell is zero. Hence, gravitational force on the point mass m_0 is zero.
24	(a) $-\frac{Gmm_0}{r_0}$
108	

$$v = -\frac{G \sum m}{r} = -\frac{G m_0}{r_0}$$



25

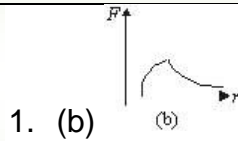
(d) 4

26

1. (a) zero

At the centre of earth, the value of g is zero so $w = mg = 0$

27



The whole space can be divided into three regions

(i) $0 < r < R_1, F(r) = 0$ (ii) $R_1 < r < R_2, F(r) = \frac{4}{3} \pi G \rho m \left(r - \frac{R_1^3}{r^2} \right)$

(iii) $R_2 < r < \infty, F(r) = \frac{4}{3} \pi G \rho m \left(\frac{R_2^3 - R_1^3}{r^2} \right)$

Here, ρ is the density of material of the sphere.

28

(b) $\sqrt{\frac{3Gm}{a^3}}$

Here $F_0 = \frac{Gm^2}{a^2}$ and $F = 2F_0 \cos 30^\circ = \frac{2Gm^2 \sqrt{3}}{a^2}$

$= \frac{\sqrt{3}Gm^2}{a^2}$ or $mr\omega^2 = \frac{\sqrt{3}Gm^2}{a^2}$ or $m \frac{a}{\sqrt{3}} \omega^2 = \frac{\sqrt{3}Gm^2}{a^2} \therefore \omega = \sqrt{\frac{3Gm}{a^3}}$

29

$$1. (b) \frac{4\pi}{3} \rho G m R^3 \left[\frac{1}{x^2} - \frac{1}{2(2x-R)^2} \right]$$

The force due to whole sphere is

$$F_1 = \frac{GMm}{x^2} = \frac{Gm}{x^2} \left(\frac{4}{3} \pi R^3 \rho \right)$$

The force due to small cutting portion is

$$F_2 = \frac{GM_1 m}{\left(x - \frac{R}{2}\right)^2} = \frac{G \left(\frac{4\pi}{3}\right) \left(\frac{R}{2}\right)^3 \rho m}{\left(x - \frac{R}{2}\right)^2}$$

$$\text{The net force is } F = F_1 - F_2 \quad \text{or} \quad = \frac{4}{3} \pi R^3 G m \rho \left[\frac{1}{x^2} - \frac{1}{2(2x-R)^2} \right]$$

30

If F_1 is the force exerted by the solid sphere, then:

$$F_1 = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}$$

If F_2 is the force exerted by the solid sphere with cavity, then:

$F_2 = F_1$ - force due to that sphere due to which cavity has been created.

$$= \frac{GMm}{4R^2} - \frac{G\left(\frac{M}{8}\right)m}{9R^2/4} = \frac{GMm}{R^2} \left(\frac{1}{4} - \frac{1}{18} \right)$$

$$= \frac{GMm}{R^2} \frac{18-4}{72} = \frac{14}{72} \frac{GMm}{R^2}$$

$$= \frac{7}{36} \frac{GMm}{R^2} = \text{NOW, } \frac{F_2}{F_1} = \frac{7}{36} \frac{GMm}{R^2} \times \frac{4R^2}{GMm} = \frac{7}{9}$$

Multiple Choice Questions

(With more than one option correct)

1. If F_1 is the magnitude of the force exerted on earth by moon and F_2 is the magnitude of force exerted on moon by earth. Which options are not correct for the statement?

- (a) $F_1 > F_2$
- (b) $F_1 = F_2$
- (c) $F_1 < F_2$
- (d) none of these

2. Out of the following, which options do not represent the gravitational force of attraction between two spherical bodies, each of mass 100 kg, if the distance between their centers is 100 m?

- (a) $6.67 \times 10^{-11} \text{ N}$
- (b) $6.67 \times 10^{-9} \text{ N}$
- (c) 6.67 N
- (d) None of these

3. Which of the following are fundamental forces?

- (a) Gravitational force
- (b) Strong nuclear force
- (c) Friction force
- (d) Weak nuclear force.

4. "A planet moves faster in its orbit". Which options do not justify the statement?

- (a) when it is farthest from the Sun.
- (b) the greater its mass.
- (c) the farther it is from its satellites.
- (d) when it is nearer the Sun.

5. Gravitation Force is a :

- (a) Conservative force
- (b) Non-conservative force
- (c) Central force
- (d) Non Central force

6. Which options are associated with Kepler's 2nd law of planetary motion?

- (a) $mvr = \text{constant}$
- (b) $dA/dt = \text{Constant}$
- (c) $MR^2 = \text{Constant}$
- (d) $mv^2/r = \text{Constant}$

7. The gravitational force between two massive spheres:

- (a) is always an attraction.
- (b) depends on how massive they are.
- (c) depends inversely on the square of the distances between them.
- (d) None of the above

8. The acceleration due to gravity decreases with:

- (a) Decreasing depth from earth surface
- (b) Decreasing height from earth surface
- (c) Increasing depth from earth surface
- (d) Increasing depth from earth surface

9. What is true about Geo-Stationary Satellite?

- (a) Revolve in the plane of equator
- (b) APPLE is a Geo-Stationary Satellite
- (c) Rotates in same direction as the earth rotates
- (d) Time period is 8 hours

10. "Point where entire weight of an object acts is....". Which options do not complete the sentence?

- (a) edge
- (b) centre of gravity
- © central point
- (d) can be anywhere in body

Answer Key

Question No.	Answer
1	Options (a), (c) and (d)
2	Options (b), (c) and (d)
3	Options (a), (b) and (d)
4	Options (a), (b) and (c)
5	Options (a) and (c)
6	Options (a) and (b)
7	Options (a), (b) and (c)
8	Options (c) and (d)

9	Options (a), (b) and (c)
10	Options (a), (c) and (d)

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UNIT-VII-- PROPERTIES OF MATTER

IMPORTANT FORMULA

1	Stress	F/A
2	Longitudinal strain	$\Delta L/L$
3	Volume strain	$-\Delta V/V$
4	Shear strain	$\Theta = x/l$
5	Young's Modulus	Stress/longitudinal strain = $(f/a)/(\Delta L/L)$
6	Bulk Modulus	$K = p/(-\Delta V/V)$
7	Compressibility	$B = 1/K = (-\Delta V/V)/p$
8	Shear Modulus or modulus of rigidity	$G = \text{Tangential stress} / \text{Shear Strain} =$

	$(f/A)/\Theta$
9 Poissons's ratio	$(-\Delta D/D)/(\Delta L/L)$
10 Elastic potential energy stored in a stretched wire	$U=1/2 F*\Delta x$
11 PE per unit volume of a wire under stress	$U'=1/2*stress*strain$
12 Isothermal Bulk Modulus	$K_i=Pressure$
13 Adiabatic bulk Modulus	$K_a=\gamma P$
14 Twisting couple per unit twist of a solid cylinder	$C=\pi\eta r^4/2l$
15 Twisting couple per unit twist of a hollow= cylinder	$C=\pi\eta(r_2^4 - r_1^4)/2l$
16 Pressure	$P=F/A$
17 Pressure exerted by fluid at a depth (hydrostatic pressure)	$P=h\rho g$
18 Gauge pressure	$P=P_a+ h\rho g$
19 Density substance	$P=M/V$
20 Relative density	RD=density of substance/density of water
21 Coefficient of viscosity	$H=F=A(dv/dx)$
22 Viscous drag or force	$F=6 \pi\eta r v$
23 Terminal velocity	$V=2/9 r^2(\rho-\sigma)g/\eta$
24 Volume of fluid flowing per unit time	$V= \pi \rho r^4/8 \eta l$
25 Reynold's number (R)	$R=v_c \rho d/\eta$
26 Volume flow rate	$(dv/dt)=av$
27 Surface Tension	$T=F/l$
28 Surface energy	$E=potential\ energy/area$
29 Excess pressure inside a soap bubble	$P=4T/r$
30 Excess pressure inside a liquid bubble	$P=2T/r$
31 Joules mechanical equivalent of heat	$J=W/Q$
32 Linear Expansion	$L'=l(1+\alpha\Delta t)$
33 Superficial Expansion	$S'=s(1+\beta\Delta t)$

34	Cubical Expansion	$V' = v(1 + \gamma \Delta t)$
35	Relation between α, β, γ	1:2:3
36	Specific heat	$C = Q/m\Delta t$
37	Thermal conductivity	$Q = KA(T_1 - T_2)t/x$
38	Heat current	$H = Q/t$
39	Krichoffs Law	$e_\lambda/a_\lambda = E_\lambda$
40	Stefan Boltzmann Law	$E = \sigma T^4$
41	Newtons law of cooling	$dT/dt = -kA(T - T_0)$
42	Wiens displacement law	$\Lambda_m = b/T$

MULTIPLE CHOICE QUESTIONS

- 1) In solids interatomic forces are
 - a) Totally repulsive
 - b) totally attractive
 - c) Both a & b
 - d) none of these

- 2) A wire of diameter 1mm breaks under a tension of 1000N. Another wire, of same material as that of the first one, but diameter 2mm breaks under a tension of
 - a) 500N
 - b) 100N
 - c) 1000N
 - d) 4000N

- 3) The breaking force of a wire of diameter D of a material is F. The breaking force for a wire of the same material of radius D is
 - a) F
 - b) 2F
 - c) F/4
 - d) 4F

- 4) Energy stored in stretching a string per unit volume is
 - a) $\frac{1}{2} \text{stress} \times \text{strain}$
 - b) stress \times strain
 - c) $\gamma(\text{Strain})^2$
 - d) $\frac{1}{2} \gamma(\text{stress})^2$

- 5) According to Hooke's Law of elasticity, if stress is increased, the ration of stress to strain
 - a) Increases
 - b) decreases
 - c) Becomes zero
 - d) remains constant

- 6) If in a wire of young modulus Y, longitudinal strain X is produced, then the value of potential energy stored in its unit volume will be
 - a) YX^2
 - b) $2YX^2$
 - c) $0.5Y^2X$
 - d) $0.5YX^2$

- 7) For a constant hydraulic stress on an object the fractional change in the object's volume and its bulk modulus are related as
 - a) $\Delta V/V \propto B$
 - b) $\Delta V/V \propto 1/B$
 - c) $\Delta V/V \propto B^2$
 - d) $\Delta V/V \propto 1/B^2$

- 8) The compressibility of water is 4×10^{-5} per unit atmospheric pressure. The decrease in volume of 100 cm^3 of water under a pressure of 100 atmospheres will be
- a) 0.4 cm^3 b) $4 \times 10^{-5} \text{ cm}^3$
c) 0.025 cm^3 d) 0.004 cm^3
- 9) A stretched rubber has
- a) Increased kinetic energy b) increased potential energy
c) Decreased kinetic energy d) decreased potential energy
- 10) The Young's modulus of a wire of length L and radius r is Y Newton per square meter. If the length is reduced to $L/2$ and radius $r/2$, its Young's modulus will be
- a) $Y/2$ b) Y
c) $2Y$ d) $4Y$
- 11) A body of density D_1 and mass M is moving downward in glycerin of density D_2 . What is the viscous force acting on it
- a) MgD_1 b) MgD_2
c) $Mg(1-D_2/D_1)$ d) $Mg(1-D_1/D_2)$
- 12) The Reynold's number for fluid flow in a pipe is independent of
- a) the viscosity of the fluid b) the velocity of the fluid
c) the length of the pipe d) the diameter of the pipe
- 13) A sphere of mass m and radius r is falling in the column of a viscous fluid. Terminal velocity attained by falling object is proportional to
- a) r^2 b) $1/r$
c) r d) $-1/r^2$
- 14) The ratio of the terminal velocities of two drops of radii R and $R/2$ is
- a) 2 b) 1
c) $1/2$ d) 4
- 15) The radii of two drops are in the ratio 3:2 their terminal velocities are in the ratio
- a) 9:4 b) 2:3
c) 3:2 d) 2:9
- 16) Two drops of equal radius coalesce to form a bigger drop. What is the ratio surface energy of bigger drop to smaller one
- a) $2^{1/2}:1$ b) 1 : 1
c) $2^{2/3}:1$ d) none of these
- 17) The rate of flow of water in a capillary tube of length l and radius r is V . The rate of flow in another capillary tube of length $2l$ and radius $2r$ for same pressure difference would be
- a) $16V$ b) $9v$
c) $8V$ d) $2V$
- 18) When a solid is converted into a gas, directly by heating, then this process is known as
- a) Boiling b) sublimation
c) Vaporization d) condensation
- 19) The density of a substance at 0° C is 10 g cm^{-3} and 100° C its density is 9.7 g cm^{-3} . The coefficient of linear expansion of the substance is

- a) 10^{-4}
c) 10^{-2}
- b) 10^{-3}
d) 10^2

20) Hailstone of 0°C falls from a height of 1 Km on an insulating surface converting whole of its kinetic energy into heat. What part of it will melt ($g = 10\text{ms}^{-2}$)

- a) $1/33$
c) $(1/3) \times 10^{-4}$
- b) $1/8$
d) all of it

21) A slab consist of two portions of different materials of same thickness and having the conductivities K_1 and K_2 . The equivalent thermal conductivity of the slab is

- a) $K_1 + K_2$
c) $2K_1K_2 / K_1 + K_2$
- b) $K_1 * K_2 / K_1 + K_2$
d) $\sqrt{K_1 + K_2}$

22) A black body is at a temperature 300K. it emits energy which is proportional to

- a) 300
c) 300^3
- b) 300^2
d) 300^4

23) The rate of dissipation of heat by a black body at temperature T is Q. What will be the rate of dissipation of heat by another body at temperature 2T and emissivity 0.25

- a) 16Q
c) 8Q
- b) 4Q
d) 4.5Q

24) A black body is heated from 27°C to 127°C . The ratio of their energies of radiations emitted will be

- a) 3:4
c) 27:64
- b) 9:16
d) 81:256

25) For an enclosure maintained at 1000K, the maximum radiation occurs at wavelength λ_m . If the temperature is raised to 2000K, the peak will shift to

- a) $\lambda_m/2$
c) $5\lambda_m/2$
- b) $3\lambda_m/2$
d) $7\lambda_m/2$

26) A metal rod at temperature of 150°C radiates energy at a rate of 20W. If it temperature is increased to 300°C then it will radiates and energy at the rate of

- a) 17.5W
c) 40.8W
- b) 37.2W
d) 68.3W

27) The sun emits a light with maximum wavelength 510nm, while another star X emits a light with maximum wavelength of 350nm. What is the ration of surface temperature of the sun and the star X

- a) 1.45
c) 0.46
- b) 0.68
d) 2.1

28) On increasing the temperature of a substance gradually its colour becomes

- a) red
c) yellow
- b) green
d) white

29) The rate of flow of liquid through an orifice of a tank does not depend upon

- a) the size of orifice
c) the height of fluid column
- b) density of liquid
d) acceleration due to gravity

30) Given that the surface tension of water is 75 dyne/cm, its density 1g/cc and angle of contact zero the height of which water rises in a capillary tube of 1mm diameter ($g = 10\text{ms}^{-2}$)

- a) 9.0cm
 b) 7.5cm
 c) 6.0cm
 d) 3.0 cm

Answers

- 1) c 2) d 3) d 4) a 5) d 6) d 7) b 8) a
 9) b 10) b 11) c 12) c 13) a 14) d 15) a 16) d
 17) c 18) b 19) a 20) a 21) b 22) d 23) b 24) d
 25) a 26) d 27) b 28) c 29) d 30) d

Explanations

1) The interatomic solid are both attractive and repulsive

2) Breaking tension $\propto r^2$
 $T_1/T_2 = (r_1/r_2)^2 = (1/2)^2 = 1/4$
 $T_2 = 4T_1 = 4 \times 1000 = 4000\text{N}$

3) Breaking force $\propto r^2$
 $F'/F = D^2/(D/2)^2 = 4$
 $F' = 4F$

4) Energy stored per unit volume = $1/2 \times \text{stress} \times \text{strain}$

5) According to hooke's Law
 stress/strain = constant

6) $U = 1/2 \times \text{stress} \times \text{strain}$
 $= 1/2 \times (Y \times \text{strain}) \times \text{strain}$

7) Bulk modulus = hydraulic stress / volumetric strain
 $B = \text{hydraulic stress} / \Delta V/V$
 $\Delta V/V = \text{hydraulic stress} \times 1/B$
 $\Delta V/V \propto 1/B$

8) Compressibility $= \Delta V/V$
 $4 \times 10^{-5} = \Delta V/100 \times 100$
 $\Delta V = 0.4 \text{ cm}^3$

9) The work done in stretching the rubber is stored as its potential energy.

10) Young's modulus depends on the nature of the material.

11) In equilibrium
 $F_v + U = Mg$
 $F_v = Mg - VD_2g \quad (D_1 = M_1/V)$
 $= Mg(1 - D_2/D_1)$

12) $R_e = \rho v D / \eta$
 R_e does not depend on the length of pipe

$$13) v = \frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta}$$

$$14) v_1/v_2 = (R/(R/2))^2 = 4$$

$$15) v_1/v_2 = (r_1/r_2)^2 = 9:4$$

16) Volume of bigger drop = volume of two smaller drop

$$\frac{4}{3} \pi R'^3 = 2 * \frac{4}{3} \pi R^3$$

$$R' = 2^{1/3} R$$

Initial surface energy

$$U_1 = 8 \pi R^2 \sigma$$

Final surface energy

$$U_2 = 4 \pi R'^2 \sigma$$

$$U_2/U_1 = 2^{-1/3}:1$$

17) Flow rate for first capillary tube

$$V = \frac{\pi r^4}{8 \eta l}$$

Flow rate for second capillary tube

$$V' = \frac{\pi (2r)^4}{8 \eta 2l}$$

$$= 8V$$

18) The direct conversion of solid into gas is called sublimation

$$19) y = \frac{\rho_0 - \rho}{\rho_0 \Delta T} = \frac{10 - 9.7}{10 * 100}$$

$$\alpha = \frac{1}{3} y = 10^{-4} \text{ c}$$

$$20) mL = Mgh$$

$$m/M = gh/L$$

$$= \frac{10 * 1000}{80 * 4.2} = 1/33$$

21) For series of combination

$$K_{eq} = \frac{d_1 + d_2}{d_1/k_1 + d_2/k_2}$$

$$(d_1 = d_2 = x \text{ say})$$

$$22) E \propto T^4$$

23) For the first body $Q = \sigma T^4$

$$\text{For the second body } Q' = \epsilon \sigma (2T)^4 = 0.25 * 16 * \sigma T^4 = 4Q$$

$$24) E_1/E_2 = ((273+27)/(273+127))^4 = 81:256$$

$$25) \lambda'_m/\lambda_m = T/T' = 1000/2000 = 1/2$$

$$\lambda'_m = \lambda_m/2$$

$$26) E_2/E_1 = ((273+300)/(273+150))^4 = 68.3$$

$$27) T_{sun}/T_{star} = \lambda_{mstar}/\lambda_{msun} = 350/510 = 0.68$$

28) By Wien's Law $\lambda_m \propto 1/T$

29) The rate of flow of liquid does not depend on size, surface area, liquid characteristic and acceleration due to gravity

$$30) h = \frac{2\sigma \cos\theta}{\rho g} = \frac{2 \times 75 \cos 0}{0.5 \times 10^{-1} \times 1 \times 1000} \text{ cm}$$

Multiple Choice Questions With 1 or more answers correct

Q1. A wire is suspended from the ceiling and stretched under the action of a weight F suspended from its other end. The force exerted by the ceiling on it is equal and opposite to the weight.

- (a) tensile stress at any cross section A of the wire is F/A .
- (b) tensile stress at any cross section is 0.
- (c) tensile stress at any cross section of the wire is $2F/A$.
- (d) tensile stress at any cross section A of the wire is F .

Q2. for an ideal liquid

- (a) the bulk modulus is infinite.
- (b) the bulk modulus is 0.
- (c) the shear modulus is infinite.
- (d) the shear modulus is 0.

Q3. when a drop splits up into number of drops

- (a) area increases
- (b) volume increases
- (c) energy is absorbed
- (d) energy is liberated

Q4. when a wire is stretched to double its length

- (a) strain is unity
- (b) stress is equal to young modulus of elasticity
- (c) its radius is half
- (d) young's modulus is equal to twice the elastic energy per unit volume

Q5. If for a liquid in a vessel force of cohesion is twice of adhesion

- (a) liquid will wet the solid
- (b) liquid will not wet the solid
- (c) the meniscus will be convex upward
- (d) the angle of contact will be obtuse

Q6. The velocity of an efflux of an ideal liquid does not depend on

- (a) area of orifice
- (b) density of liquid
- (c) area of cross section of the vessel
- (d) depth of the point below the free surface of the liquid

Q7. A piece of wood is floating in water kept in a bottle. The bottle is connected to an air pump. When more air is pushed into the bottle from the pump.

- (a) the thrust of air will increase
- (b) the thrust of air will remain unchanged
- (c) the thrust of the water will decrease
- (d) the wood piece will rise a little

Q8. Which of the following statements (s) is / are correct about a soap bubble?

- (a) work done in forming the bubble of radius R and surface tension T is $8 R^2 T \pi$
- (b) work done in doubling the radius of the bubble of radius R and surface tension is $12 R^2 T \pi$.
- (c) pressure inside the bubble is double than inside the drop of same radius and liquid.
- (d) pressure inside the bubble is lesser than outside it.

Q9. A copper and a steel wire of the same diameter are connected end to end. A deforming force F is applied to this composite wire which causes a total elongation of 1 cm. The two wires will have

- (a) the same stress
- (b) different stress

(c) the same strain

(d) different strain

Q10. Pressure exerted by a liquid depend upon

(a) depth of liquid

(b) density of air

(c) density of liquid

(d) volume of liquid

Answers

Q1. (a,d)

Q2. (a,d)

Q3. (a,c)

Q4. (a,b &d)

Q5. (b,c&d)

Q6. (a,b&c)

Q7. (a,b,c,&d)

Q8. (a,c)

Q9. (a,d)

Q10. (a,c)

MORE THAN ONE ANSWER

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- (d) different strain

Q10. Pressure exerted by a liquid depends upon

- (a) depth of liquid
- (b) density of air
- (c) density of liquid
- (d) volume of liquid

Answer Key

1	2	3	4	5	6	7	8	9	10
(a) (d)	(a) (d)	(a) (c)	(a)(b)(d)	(b)(c) (d)	(a)(b)(c)	(a)(b)(c) (d)	(a) (c)	(a) (d)	(a) (c)

ANSWER

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Q9. (a) the same stress

(d) different strain

Q10. (a) depth of liquid

(c) density of liquid

UNIT VIII – THERMODYNAMICS

GIST AND FORMULAE OF THERMODYNAMICS

Thermodynamics: - It is the branch of science that deals with the concepts of heat and temperature and the inter conversion of heat and other forms of energy.

Thermodynamic System: - An assembly of a very large number of particles having a certain value of pressure volume and temperature is called a thermodynamic system.

Surroundings: - Everything outside the system which can have a direct effect on the system is called its surrounding.

Thermodynamic variables: - The quantities like pressure (P), volume (V), and temperature (T) which help us to study the behavior of a thermodynamic system or call thermodynamic variables.

Equation of state:-The mathematical relation between the pressure volume and temperature of a thermodynamic system is called its equation of state. For example, the equation of state of n moles of an ideal gas can be written as $PV = nRT$.

Thermal equilibrium:-Two systems are in thermal equilibrium with each other if they have the same temperature.

Thermodynamic Equilibrium:-A system is said to be in the state of Thermodynamic Equilibrium if the microscopic variables describing the thermodynamic state of the system do not change with time a system in a state of Thermodynamic Equilibrium Oasis mechanical thermal and chemical equilibria simultaneously.

State variables:-The microscopic quantities which are used to describe the equilibrium states of a thermodynamic system are called state variables. The value of a state variable depends only on the particular state do not on the path used to attend the state pressure (P) , volume (V) , temperature (T) and mass (m) are state variables. Heat (Q) work (W) are not state variables.

Zeroth law of thermodynamics:-If two system A and B are in thermal equilibrium with a third system C, then A and B are in thermal equilibrium with each other. According to this lower temperature is a physical quantity which has the same value for all systems which are in a thermal equilibrium with each other.

Internal energy the internal energy of a system is the sum of molecular Kinetic and potential energy in the frame of reference related to which the Centre of mass of the system is at rest. It does not include the overall kinetic energy of the system .It is a state variable denoted by U.

Quasi static process: - A Quasi static process is an infinitely slow process is such that system remains in thermal and mechanical equilibrium with the surroundings throughout in such a process, the pressure and temperature of the surroundings can differ from those of the system only infinitesimally.

Isothermal process:-A process in which temperature remain constant is called isothermal process for such a process, $PV = \text{constant}$ or $P_1V_1=P_2V_2$.

Adiabatic process:-A process in which thermal insulated system neither loses nor gains heat from the surroundings is called adiabatic process.

Equations of state for adiabatic processes are:

$$\begin{aligned} \text{i. } & PV^\gamma = \text{constant} & \text{or} & & P_1V_1^\gamma = P_2V_2^\gamma \\ \text{ii. } & TV^{\gamma-1} = \text{constant} & \text{or} & & T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \\ \text{iii. } & p^{\gamma-1}/T^\gamma = \text{constant} & \text{or} & & P_1^{\gamma-1}/T_1^\gamma = P_2^{\gamma-1}/T_2^\gamma \end{aligned} \quad \gamma = \frac{C_p}{C_v}$$

Isobaric process: - A process in which volume remains constant is called isobaric process process in which volume remains constant is called isochoric process. For such a process,

$$V/T = \text{constant or } V_1/T_1 = V_2/T_2 .$$

Isochoric process: - A process in which pressure remains constant is called isochoric process. For a such a process,

$$P/T = \text{constant or } P_1/T_1 = P_2/T_2$$

Diagram:-A graphical representation of the state of a system with the help of 2000 methyl variables is called indicator diagram of the system. The graph between pressure (P) and volume (V) is called P-V diagram.

Work done during the expansion of a gas:-When the volume of a gas changes from V_1 to V_2 , the work done is $W = \int_0^W dW = \int_{v_1}^{v_2} PdV$ = area enclosed between the PV curve and the volume axis.

Work done during a cycle process: - From the P-V diagram,

Work done per cycle = area of the loop representing the cycle

- i. If the loop is traced *clockwise*, the work done is positive and work is done by the system.
- ii. If the loop is traced *anticlockwise*, the work done is negative and work is done on the system.

First law of thermodynamics:-It is just that it if heat dQ is given to a system, a part of it is used in increasing the internal energy of an amount dU and the remaining energy is used in doing the external work dW . It is just a statement of the law of conservation of energy. Thus

$$dQ = dU + dW \quad \text{or} \quad dQ = dU + PdV$$

Sign conventions used:-

- i. Heat absorbed by a system is positive and heat given out by a system is negative.
- ii. Increase in internal energy of a system is positive and decrease in internal energy of a system is negative.
- iii. Work done by a system is positive and work done on a system is negative.

Work done in an isothermal process: - work done when 1 mole of a gas expands isothermally,

$$W = 2.303 RT \log_{10} \frac{V_2}{V_1}$$

Work done in an adiabatic process: - Work done when 1 mole of a gas expands adiabatically and its temperature falls from T_1 to T_2 ,

$$W_{adi} = \frac{R}{\gamma-1} [T_1 - T_2] = \frac{1}{\gamma-1} [P_1 V_1 - P_2 V_2].$$

Dulong and petit's law: - Near the room temperature, the molar specific heat of most of the solids at constant volume is equal to $3R$ or $6 \text{ cal mol}^{-1} \text{ K}^{-1}$ or $25 \text{ J mol}^{-1} \text{ K}^{-1}$. This statement is known as Dulong and petit's law.

Molar specific heat of a gas at constant volume (c_v): - It is defined as the amount of heat required to raise the temperature of 1 mole of the gas through 1°C at constant volume.

If c_v is specific heat of a gas for 1 g at constant volume and M is its molecular weight, then molar specific heat at constant volume, $C_V = M c_v$

Molar specific heat of a gas at constant pressure (c_p): - It is defined as the amount of heat required to raise the temperature of 1 mole of the gas through 1°C at constant pressure.

Thus, $C_p = M c_p$

Relation between two specific heats of a gas: - Specific heat of a gas at constant pressure is greater than the specific heat at constant volume.

For 1 mole of a gas:

- i. $C_p - C_v = R$ (Where C_p, C_v are in units of work)
- ii. $C_p - C_v = R/J$ (Where C_p, C_v are in units of heat)
Where R is universal gas constant for 1 mole of a gas.

For 1g of a gas:

- i. $C_p - C_v = r$ (where C_p, C_v are in units of work)
- ii. $C_p - C_v = R/J$ (where C_p, C_v are in units of heat)
Where $r = R/M =$ gas constant for 1 g of a gas.

Clearly, heat lost or gained by n moles of a gas,

- i. $Q = n C_p \Delta T$ (At constant pressure)
- ii. $Q = n c_v \Delta T$ (At constant volume)

Where $n =$ number of moles of gas $= \frac{\text{Mass of gas}}{\text{molecular mass}}$.

Heat engine: - It is a device which converts continuously to heat energy into mechanical energy in a cyclic process. It essentially consist (1) of a source of heat (2) sink of heat and (3) a working substance.

Efficiency of heat engine: - It is the ratio of useful work done (W) by the engine per cycle to the heat energy (Q_1) absorbed from the source for cycle.

$$\eta = \frac{\text{Work output}}{\text{Heat input}} = W/Q_1 = Q_1 - Q_2/Q_1 = 1 - Q_2/Q_1 \text{ Where } Q_2 \text{ is the heat rejected to the sink.}$$

Second law of thermodynamics:-

- i. **Kelvin-Planck statement:**-It is impossible to construct engine, which will produce no effect other than extracting heat from a Reservoir and performing an equivalent amount of work.
- ii. **Clausius statement:** - It is impossible for self acting machine unaided by an external agency, to transfer heat from a body to another at higher temperature.

Reversible process: - A process which can be made to proceed in the reverse direction by variation in its conditions so that any change occurring in any part of the direct process is exactly reversed in the corresponding part of the reverse process is called a reversible process.

Irreversible process: - A process which cannot be made to proceed in the reverse direction is called an Irreversible process.

Carnot engine: - It is an ideal heat engine which is based on Carnot's reversible cycle. Its working consists of 4 Steps viz. Isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression. The efficiency of Carnot engine is given by

$$\eta = 1 - Q_2/Q_1 = 1 - T_2/T_1$$

Where T_1 and T_2 are the temperatures of source and sink respectively.

Carnot theorem: - It states that

- i. no Engine working between two given temperatures can have efficiency greater than that of the Carnot engine working between the same to temperatures and
- ii. The efficiency of a Carnot engine is independent of the nature of the working substance.

Refrigerator: - It is a heat engine working in the reverse direction. Here a working substance absorbs heat Q_2 from the sink at temperature T_2 . An external agency does work (W) on the working substance. A larger amount of heat Q_1 is rejected to source at a higher temperature T_2 .

$$Q_1 = Q_2 + W.$$

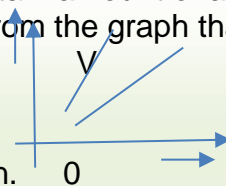
Coefficient of performance: - It is defined as the ratio of the amount of heat (Q_2) removed per cycle from the contents of the refrigerator to the work done (W) by the external agency to remove it.

$$\beta = Q_2/W = Q_2/Q_1 - Q_2 = T_2/T_1 - T_2.$$

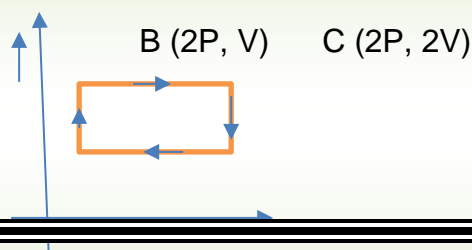
MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. First law of thermodynamics is the law of conservation of
a) Mass b) linear momentum c) energy d) angular momentum.
2. Internal energy of an isolated system
a) Increases b) decreases c) remains the same d) none of these.
3. The change in internal energy of a system in a cyclic process is
a) Positive b) negative c) zero d) may be positive or negative.
4. An ideal Carnot engine whose efficiency is 40% receives heat at 500K. If the efficiency is to be 50%, the intake temperature for the same exhaust temperature is
a) 600K b) 700K c) 800K d) 900K
5. An engine has an efficiency of 1/6. When the temperature of sink is reduced by 62°C, its efficiency is doubled. The temperature of the source is
a) 37°C b) 62°C c) 99°C d) 124°C
6. Mathematical form of first law of thermodynamics is
7. $dQ = dU - PdV$ b) $dQ = dU + PdV$ c) $dU + dQ = PdV$ d) $dQ + PdV = dU$.
8. Work done in isothermal process is
a) $\eta RT \log \frac{V_1}{V_2}$ b) $\eta RT \log \frac{V_2}{V_1}$ c) $\eta RT \log \frac{P_1 V_1}{P_2 V_2}$ d) $\eta R(T_2 - T_1) \log \frac{V_1}{V_2}$.
9. Work done in adiabatic process is
a) $\frac{RT_1}{\gamma-1}$ b) $\frac{RT_2}{\gamma-1}$ c) $\frac{R(T_1-T_2)}{\gamma-1}$ d) $R(\gamma-1)(T_2 - T_1)$

10. The efficiency of Carnot engine depends on
 a) Nature of working substance b) temperature of the source c) temperature of the sink d) temperature of the source and sink.
11. A carnot engine whose sink is at 300k has an efficiency of 40%. By how much should the temperature of source be increased so as to increase its efficiency by 50% of original efficiency?
 a) 250 K b) 275 K c) 325 K d) 380 K
12. During the adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio $\frac{C_P}{C_V}$ for the gas is
 a) 2 b) $\frac{5}{3}$ c) $\frac{4}{3}$ d) $\frac{3}{2}$
13. A thermos flask made of stainless steel contains several tiny lead shots. If the flask is quickly shaken up and down several times, the temperature of lead shots
 a) Increase by adiabatic process b) increase by isothermal process
 c) decrease by adiabatic process d) first increase and then decrease.
14. Which of the following statements is correct?
 a) Work done is state dependent but not path dependent
 b) Internal energy of a gas depends only on the state of gas
 c) Area under P – V graph equals heat supplied in any process
 d) In an isothermal process, change in internal energy is maximum.
15. When you make ice cubes, the entropy of water
 a) Increase b) decrease c) does not change
 d) may either increase or decrease depending on the process used.
16. One mole of an ideal gas at an initial temperature of T K does 6R joules of work adiabatically, If $\gamma = \frac{C_P}{C_V} = \frac{5}{3}$, the final temperature of gas will be
 a) (T + 4) K b) (T-4) K c) (T+2) K d) (T-2) K
17. The work done of 146 KJ is performed in order to compress one kilo mole of a gas adiabatically and in this process, the temperature of the gas increases by 7°C. The gas is (R = 8.3J) mol⁻¹K⁻¹)
 a) Monoatomic b) diatomic c) triatomic d) a mixture of monoatomic and diatomic
18. During isothermal expansion, a confined ideal gas does 150J of work against its surroundings. This implies that
 a) 150J of heat has been added to the gas
 b) 300J of heat has been added to the gas
 c) no heat is transferred because the process is isothermal
 d) 150 J of heat has been removed from the gas
19. In a given process of an ideal gas, dW = 0 and dQ < 0. Then for the gas
 a) The temperature will decrease
 b) the volume will increase
 c) the pressure will remain constant
 d) the temperature will increase
20. The volume (V) versus temperature (T) graph for a certain amount of a perfect gas at two pressure P₁ and P₂ are shown in the figure. It follows from the graph that
 a) P₁ > P₂
 b) P₁ < P₂
 c) P₁ = P₂
 d) Information is insufficient to draw any conclusion.



21. An ideal monatomic gas is taken round the cycle ABCDA as shown in the P-V diagram.



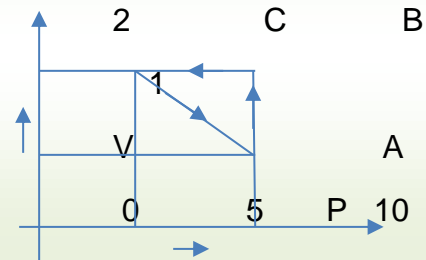
A (P, V) B (P, 2V)

0 V

The work done during the cycle is

- a) PV b) 2PV c) PV/2 d) zero.

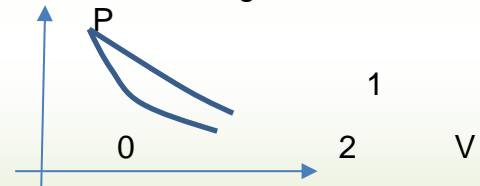
22. An ideal gas is taken through the cycle A→B→C→A as shown in the figure. if the net heat



Supplied to the gas in the cycle is 5J, the work done by the gas in the process C→A is

- a) -5J b) -10J c) -15J d) -20J

23. P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to



- a) He and O₂ b) O₂ and He c) He and Ar d) O₂ and N₂

24. An ideal gas initially at P₁, V₁ is expanded to P₂, V₂ and then compressed adiabatically to the same volume V₁ and pressure is P₃. If W is the net work done by the gas in complete process, which of the following is true?

- a) W > 0; P₃ > P₁ b) W < 0; P₃ > P₁
 c) W > 0; P₃ < P₁ d) W < 0; P₃ < P₁

25. A monatomic ideal gas, initially at temperature T₁ is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T₂ by releasing the Piston suddenly. L₁ and L₂ at the lengths of the gas column before and after expansion respectively, then T₁/T₂ is given by

- a) (L₁/L₂)^{2/3} b) L₁/L₂ c) L₂/L₁ d) (L₂/L₁)^{2/3}

26. One mole of a monatomic gas is heated at a constant pressure of 1 atmosphere from 0K to 100K. If the gas constant R = 8.32 J/mol K, the change in internal energy of the gas is approximately

- a) 2.3J b) 46J c) 8.67 × 10³J d) 1.25 × 10³J

27. An ideal gas heat engine is of working between 227°C and 127°C. It absorbs 10⁴ J of heat at the higher temperature. The amount of heat converted into work is

- a) 2000 J b) 4000 J c) 8000 J d) 5600 J

28. To decrease the volume of a gas by 5% at a constant temperature, pressure should be

- a) Decrease by 5.26% b) Increase by 5.26%
 c) Decrease by 11% d) Increase by 11%

29. The efficiency of a Carnot engine operating with reservoir temperature of 100°C and -23°C will be
 a) $\frac{100+23}{100}$ b) $\frac{100-23}{100}$ c) $\frac{100+23}{373}$ d) $\frac{100-23}{373}$
30. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 litre. Taking the initial temperature to be T_1 , the work done in the processes is
 a) $\frac{9}{8}RT_1$ b) $\frac{3}{2}RT_1$ c) $\frac{15}{8}RT_1$ d) $\frac{9}{2}RT_1$

ANSWERS OF MULTIPLE CHOICE QUESTIONS

1. (c) 2. (c) 3. (c) 4. (a) 5. (c) 6. (b) 7. (b) 8. (c) 9. (c) 10. (c)
11. (a) $\eta = 40\% = \frac{2}{5}$ $\eta = 1 - \frac{T_2}{T_1}$ or, $\frac{2}{5} = 1 - \frac{300}{T_1}$ or, $\frac{300}{T_1} = 1 - \frac{2}{5} = \frac{3}{5}$ so, $T_1 = 500 \text{ K}$. Now $\eta' = \left(\frac{2}{5} + 50\% \text{ of } \frac{2}{5}\right) = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$ and $\eta' = 1 - \frac{T_2'}{T_1}$ or, $\frac{T_2'}{T_1} = 1 - \eta'$
 $\eta' = 1 - \frac{3}{5} = \frac{2}{5}$ or, $T_1' = \frac{5}{2} \times T_2 = \frac{5}{2} \times 300 = 750 \text{ K}$ so, increase in Temp 250 k.
12. (d) During adiabatic process, $P\gamma^{-1} \propto T^\gamma$ or, $P \propto T^{\frac{\gamma}{\gamma-1}}$ given, $P \propto T^3$,
 so, $\frac{\gamma}{\gamma-1} = 3$ or, $\gamma = \frac{3}{2}$.
13. (a) When lead shots fall from one end to another end of the flask, mechanical energy of lead shots is converted in to heat energy. The heat energy increases the temperature of lead shots during adiabatic process.
14. (b) The internal energy of the gas changes only if the state of the gas changes.
15. (b) $dS = \frac{dQ}{T}$ when ice is formed, heat is extracted from the water i.e. dQ is negative.
16. (b) Work done during adiabatic process, $w = \frac{R}{\gamma-1}(T_1 - T_2)$ or $6R = \frac{R}{\frac{5}{3}-1}(T - T_2) = \frac{3R}{2}(T - T_2)$ so, $T_2 = (T - 4)K$.
17. (b) $W = \frac{\mu R}{\gamma-1}(T_1 - T_2)$ so, $\gamma - 1 = \frac{\mu R}{W}(T_1 - T_2) = \frac{8.3}{146} \times 7 = 0.4$ so, $\gamma = 1.4$ (Diatom)
18. (d) $\Delta Q = \Delta u + W$ so during isothermal process, $\Delta u = 0$ so, $\Delta Q = W + 150J$.
19. (a) By first law of thermodynamics, $dQ = du + dW$ as $dW = 0$ and $dQ < 0$, so, $du < 0$
 But for an ideal gas $u \propto T$ so, $dT < 0$ hence the temperature of the gas decreases.
20. (a) At a given temperature T , $P_1V_1 = P_2V_2$ from the graph, $V_1 < V_2$ and $P_1 > P_2$.
21. (a) Work done in the cyclic process, Area of the loop ABCD = $(2P - P) \times (2V - V) = PV$
22. (a) For the cyclic process, $\Delta Q = \Delta W = W_{AB} + W_{BC} + W_{CA}$ or, $5 = 10(2 - 1) + 0 + W_{CA}$
 $W_{CA} = 5 - 10 = -5J$
23. (b) Slope of an adiabatic curve, $\frac{dP}{dV} = -\gamma \frac{P}{V}$ slope $\propto \gamma$.
 From the given graph, slope of plot 2 $>$ slope of plot 1. $\gamma_2 > \gamma_1$ So, plot 1 should correspond to diatomic O_2 ($\gamma = 1.4$) and plot 2 should correspond to monoatomic He (large $\gamma = 1.67$).
24. (b) The slope of an adiabatic curve is γ times the slope of an isothermal curve at any given point. In the figure, AB is an isotherm and BC is an adiabat.
25. (d) For an adiabatic expansions, $TV^{\gamma-1} = \text{constant}$ or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ for a monoatomic gas $\gamma = 5/3$ If A is the area of cross section of the piston, then $\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{AL_2}{AL_1}\right)^{\frac{5}{3}-1} = \left(\frac{L_2}{L_1}\right)^{2/3}$
26. (d) For a monoatomic gas, $C_v = \frac{3}{2}R$ or, $du = C_v dT = \frac{3}{2}RT = \frac{3}{2} \times 8.32 \times (100 - 0) = 1.25 \times 10^3 J$.
27. (a) $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273+127}{273+227} = 1 - \frac{400}{500} = \frac{1}{5}$ so $W = \eta Q_1 = \frac{1}{5} \times 10^4 = 2000J$.

28. (b) At constant temperature, $PV = \text{constant}$. When V is decreased by 5%, then P has to be increased say dP so, $(P + dP)(V - 5/100V) = PV$ or, $dP = 5.26\%$.

29. (c) $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273-23}{273+100} = 1 - \frac{250}{373} = \frac{373-250}{373} = \frac{123}{373} = \frac{100+23}{373}$.

30. (a) for atomic gas He gas $\gamma = \frac{5}{3}$. so, $V_1 = 5.6 \text{ Litre}$, $T_1 = 273K$, $P_1 = 1 \text{ atm}$, $V_2 = 0.7 \text{ Litre}$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \text{ or, } T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 \left(\frac{5.6}{0.7}\right)^{\frac{5}{3}-1} \text{ or, } T_1 8^{\frac{2}{3}} = 4T_1 \text{ so, } \eta = \frac{5.6}{22.4} = \frac{1}{4}$$

$$W_{adi} = \frac{\eta R(T_2 - T_1)}{\gamma - 1} = \frac{1/4 R(4T_1 - T_1)}{(5/3) - 1} = \frac{9}{8} RT_1.$$

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

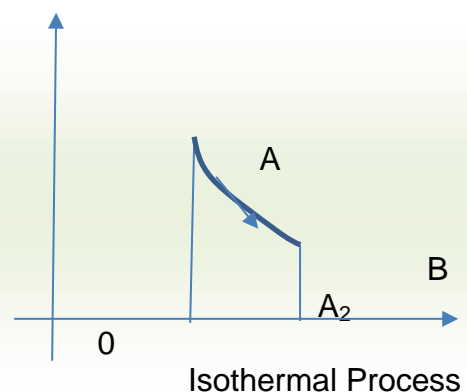
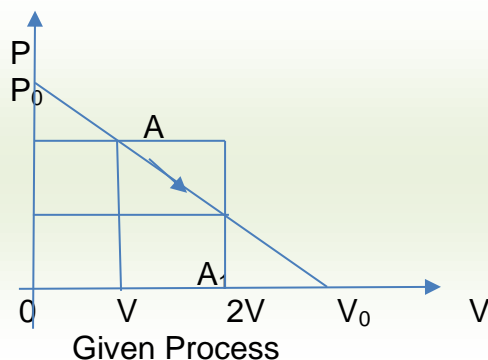
1. An ideal gas is taken from the state A (pressure P , volume V) to the state B (pressure $P/2$, volume $2V$) along a straight line path in the P - V diagram. Select the correct statement(s) from the following:
 - a) the work done by the gas in the process A to B exceeds the work that would be done by it, if the system were taken from A to B along an isotherm
 - b) in the T - V diagram, the path AB becomes a part of a parabola
 - c) in the P - T diagram, the path AB becomes a part of a hyperbola
 - d) in going from A to B, the temperature T of the gas first increases to a maximum value and then decreases.
2. During the melting of a slab of ice at 273 K at atmospheric pressure,
 - a) positive work is done by the ice-water system on the atmosphere
 - b) positive work is done on the ice-water system by the atmosphere
 - c) the internal energy of the ice-water system increases
 - d) the internal energy of the ice-water system decreases.
3. 70 calories are required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required (in calories) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is

a) 30 b) 50 c) 70 d) 90
4. For an ideal gas
 - a) the change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_V(T_2 - T_1)$, where C_V is the molar specific heat at constant volume and n , the number of moles of the gas
 - b) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process
 - c) no heat is added or removed in an adiabatic process
 - d) the internal energy does not change in an isothermal process.

5. Two cylinders A and B fitted with Pistons contain equal amounts of an ideal diatomic gas at 300k. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 k, then the rise in temperature of the gas in B is
- a) 18 k b) 30 k c) 50 k d) 42 k
6. Two identical containers A and B with frictionless piston contain the same ideal gas at the same temperature and the same volume V . The mass of the gas in A is m_A and that in B is m_B . The gas in each cylinder is now allowed to expand isothermally to the same final volume $2V$. The changes in the pressure in A and B are found to be P and $1.5 p$ respectively. Then
- a) $2m_A = 3m_B$ b) $3m_A = 2m_B$ c) $4m_A = 9m_B$ d) $9m_A = 4m_B$
7. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA as shown in the figure. Its pressure at A is P_0 . Choose the correct option(s) from the following:
- a) internal energies at A and B are the same
 b) work done by the gas in process AB is $P_0V_0 \ln 4$
 c) Pressure at C is $P_0/4$
 d) Temperature at C is $T_0/4$
-
8. Which of the process described below are irreversible?
- a) The increase in temperature of an iron rod by hammering it
 b) A gas in small container at a temperature T_1 is brought in contact with a big reservoir at a higher temperature T_2 which increases the temperature of the gas.
 c) A quasi – static isothermal expansion of an ideal gas in cylinder fitted with a frictionless piston.
 d) An ideal gas is enclosed in a piston cylinder arrangement with adiabatic walls. A weight w is added to the piston, resulting in compression of gas.
9. An ideal gas under goes isothermal process from some initial state I to final state f. Chose the correct alternatives.
- a) $dU = 0$ b) $dQ = 0$ c) $dQ = dU$ d) $dQ = dW$

ANSWERS OF MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

1. (a), (b) and (d) are correct
 Work done = Area under $P -$ diagram, Area $A_1 >$ Area A_2 . So, $W_{\text{given process}} > W_{\text{isothermal}}$
 So (a) is correct answer.



If P_0 and V_0 are the intercepts on P and V axis the equation of line AB will be

$$P = -\frac{P_0}{V_0}V + P_0 \text{ or, } \frac{\mu RT}{V} = -\frac{P_0}{V_0}V + P_0 \text{ or, } T = \frac{1}{\mu R} \left(\frac{P_0}{V_0} V^2 + P_0 V \right) \text{ as } T \propto V^2$$

so, $T - v$ graph will be a parabola. Hence option (b) is correct.

$$\text{Again } P = -\frac{P_0}{V_0} \frac{\mu RT}{P} + P_0 \text{ or, } \frac{V_0 P^2}{\mu R P_0} = -T + \frac{V_0}{\mu R} P \text{ or, } T = \frac{V_0}{\mu R P_0} P^2 + \frac{V_0}{\mu R} P \text{ as } t \propto P^2$$

As $P - T$ graph is a parabola not a hyperbola. Hence option © is incorrect.

$$T = \frac{V_0}{\mu R} \left(-\frac{1}{P_0} P^2 + P \right) \text{ so, } \frac{dT}{dP} = \frac{V_0}{\mu R} \left(-\frac{2P}{P_0} + 1 \right) = 0 \text{ or, } P = \frac{P_0}{2} \text{ so, } \frac{d^2T}{dP^2} = -\frac{2V_0}{\mu R P_0} < 0$$

So T has a maximum value. $T \propto PV$ or, $(PV)_A = (PV)_B$ or, $T_A = T_B$. So in going from A to B, T first increases to a maximum and then decreases to original value. Hence option (d) is correct.

2. (b) and (c) When ice slab melts at 273K, its volume decreases. So, -ve work is done by ice-water system on the atmosphere or +ve work is done by the atmosphere on ice-water system. Hence (b) is correct.

From first law of thermodynamics, $dQ = du + dW$ or $du = dQ - dW$ so, dQ is +ve as ice absorbs heat during melting. Also, dW is -ve. Hence du will be internal energy of ice-water system increase. Hence option © is correct.

3. (b) is correct At constant pressure, $Q_1 = \mu C_P dT$ and at constant volume, $Q_2 = \mu C_V dT$

$$\frac{Q_2}{Q_1} = \frac{C_V}{C_P} = \frac{1}{\gamma} \text{ So, } Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4} = 50 \text{ Cal.}$$

4. (a), (b), (c) and (d) are correct. (a) $\Delta u = \mu C_V \Delta T = \mu C_V (T_2 - T_1)$, for all process.

(b) In an adiabatic process, $\Delta Q = 0$ so, $\Delta u = -\Delta w$

$$\text{or, } |\Delta u| = |\Delta w| \text{ (c) In an isothermal process, } \Delta T = 0$$

(d) In an adiabatic process, $\Delta Q = 0$. Hence all options are correct.

5. (d) As the piston of A is free to move, heat is supplied at constant pressure.

So, $dQ_A = \mu C_P dT_A$. As the piston of B is fixed, heat is supplied at constant volume.

$$dQ_B = \mu C_V dT_B \text{ but } dQ_A = dQ_B \text{ or } \mu C_P dT_A = \mu C_V dT_B \text{ so, } dT_B = \left(\frac{C_P}{C_V} \right) dT_A = \gamma dT_A$$

$dT_B = 1.4 \times 30 \text{ K}$ (for a diatomic gas, $\gamma = 1.4$) = 42.0 = 42 K.

6. (b) $\Delta P = P_i - p_f = \frac{\mu RT}{V} - \frac{\mu RT}{2V} = \frac{\mu RT}{2V} = \frac{mRT}{2MV}$ so, $\Delta P \propto m$ so, $\frac{m_A}{m_B} = \frac{\Delta P_A}{\Delta P_B} = \frac{P}{1.5P} = \frac{2}{3}$ or, $3m_A = 2m_B$

7. (a), (b), (c) and (d) Process A → B is isothermal, so $U_A = U_B$ so, $W_{AB} = \mu RT \ln \frac{V_2}{V_1}$

$$W_{AB} = RT_0 \ln \frac{4V_0}{V_0} = P_0 V_0 \ln 4 \text{ [} P_0 V_0 = RT_0 \text{ for } \mu = 1 \text{] now } \frac{P_0 V_0}{T_0} \text{ (at A)} = \frac{P_B 4V_0}{T_0} \text{ (at B)}$$

so, $P_B = \frac{P_0}{4}$. Process B → C is isobaric, so, $p_C = P_B = \frac{P_0}{4}$.

Process C → A is isochoric, so, $\frac{p}{T} = \text{constant}$ so, $T_C = \frac{T_0}{4}$.

8. (a), (b) and (d). (a) In this case internal energy of the rod is increased from external work done by hammer which in turn increases its temperature. So, the process cannot be retraced itself.

(b) In this process energy in the form of heat is transferred to the gas in the small container by big reservoir at temperature T_2 .

(c) As the weight is added to the cylinder arrangement in the form of external pressure hence, it cannot be reversed back itself.

9. According to first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$ but $\Delta U \propto \Delta T$ so, $\Delta U = 0$ as $\Delta T = 0$. $\Delta Q = \Delta W$, i.e. heat supplied in an isothermal change is used to do work against external surroundings or, if the work is done on the system then equal amount of heat energy will be liberated by the system.

UNIT IX – KINETIC THEORY OF GASES
GIST AND FORMULAE OF KINETIC THEORY OF GASES

- **Pressure exerted by a gas** : It is due to continuous collision of gas molecules against the walls of the container and is given by the relation

$$P = \frac{Mc^2}{3V} = \frac{1}{3} \rho c^2 \text{ where } c \text{ is the rms velocity of gas molecules.}$$

- **Average K.E. per molecule** of a gas $\frac{1}{2} mC^2 = \frac{3}{2} k_B T$. It is independent of the mass of the gas but depends upon the temperature of the gas.
- **Absolute zero** : It is that temperature at which the root mean square velocity of the gas molecules reduces to zero.
- **Different types of speed of gas molecules**

(i) Most probable speed $v_{mp} = \sqrt{\frac{2k_B T}{m}}$

$k_B \rightarrow$ Boltzmann's Constant

- (ii) Mean speed

$$v_{\text{mean}} = \frac{v_1 + v_2 + \dots + v_n}{n} = \sqrt{\frac{8k_B T}{m\pi}}$$

$m =$ mass of one molecule of gas

$T =$ Temperature of gas

- (iii) Root mean square speed

$$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_n^2}{n}} = \sqrt{\frac{3k_B T}{m}}$$

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- (iii) Root mean square speed

$$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_n^2}{n}} = \sqrt{\frac{3k_B T}{m}}$$

(iv) The number of degrees of freedom = total number of independent coordinates required to describe completely the position and configuration of a system. For monoatomic gases, $f = 3$

For diatomic gases, $f = 5$

For linear triatomic gas molecules, $f = 7$

For non-linear triatomic gas molecules, $f = 6$

- According to the **law of equipartition of energy**, for any dynamical system in thermal equilibrium, the total energy is distributed equally amongst all the degrees of freedom. The average energy associated with each molecule per degree of freedom = $\frac{1}{2} k_B T$, where k_B is Boltzmann constant and T is temperature of the system.
- **Mean free path** of gas molecules is the average distance travelled by a molecule between two successive collisions. It is represented by λ .

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

where d = diameter of molecule and n = number of molecules per unit volume of the gas.

$$\text{also } \lambda = \frac{k_B T}{\sqrt{2}\pi d^2 p}$$

where k_B is Boltzmann constant; p is pressure and T is temperature of the gas.

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. Give the dimensional formula of thermal conductivity.

- (a) $[k]=[MLT^{-1}K^{-1}]$ (b) $[k]=[M^{-1}LT^{-1}K^{-1}]$ (c) $[k]=[MLT^{-1}K^{-2}]$ (d) $[k]=[MLT^{-2}K^{-2}]$

2. The r.m.s. speed of the molecules of an enclosed gas is x . what will be the r.m.s. speed, if the pressure of the gas is doubled but the temperature is kept constant?

- (a) 100 K (b) 150K (c) **400 K** (d) 300K

3. One mole of mono atomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$) what will be the value of γ for the mixture?

- (a) 1 (b) **1.5** (c) 2 (d) 3.5

4. The mean free path of nitrogen molecules at $27^\circ C$ is 3×10^{-7} m/s. if the average speed of nitrogen molecules at the same temperature is 600 m/s then what will be the collision frequency?

- (a) $3 \times 10^9/\text{sec}$ (b) $2.5 \times 10^9/\text{sec}$ (c) $5 \times 10^9/\text{sec}$ (d) **$2 \times 10^9/\text{sec}$**

5. Which is the relation between mean translational K.E. per unit volume E and the pressure P of a perfect gas .

- (a) $P = \frac{2E}{3V}$ (b) $P = \frac{3E}{2V}$ (c) $P = \frac{3E}{5V}$ (d) $P = \frac{3E}{V}$

6. If two gases of molecular weights M_1 and M_2 are at same pressure and temperature, write the ratio of their r.m.s. speed.

- (a) $\sqrt{M_2} : \sqrt{M_1}$ (b) $\sqrt{M_1} : \sqrt{M_2}$ (c) $\sqrt{M_2} : \sqrt{M_1}$ (d) none of these.

7. The degree of freedom for Linear atomic. (For Linear atomic $N=3, R=2$ $f=3 \times 3 - 2 = 7$)

- (a) 3 (b) 4 (c) **7** (d) 5

8. What is the ratio of velocities $V_{av} : V_{mp} : V_{rms}$ in context of kinetic theory of gases ?

- (a) **1.6:1.41: 1.73** (b) 1.41:1.6: 1.73 (c) 1.41:1.6: 1.73 (d) 1.6:1.73: 1.41

9. The r.m.s. speed of the molecules of an enclosed gas is P . what will be the r.m.s. speed, if the pressure of the gas is doubled but the temperature is kept constant?

- (a) 0 (b) 3×10^8 m/s (c) **P** (d) 330m/s

10. One mole of mono atomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$) what will be the value of γ for the mixture?

- (a) 1.3 (b) 1.4 (c) **1.7** (d) **1.5**

11. In a gas, 5 molecules have speed 150 m/s, 170 m/s, 170 m/s, 180 m/s, 190 m/s. What is the ratio of V_{rms} to V_{mean} ?

- (a) 0 (b) **1** (c) 2 (d) 1/2

12. The mean free path of nitrogen molecules at 27°C is 3×10^{-7} m. If the average speed of nitrogen molecules at the same temperature is 600 m/s then what will be the collision frequency?

- (a) 0sec (b) **2×10^9 /sec** (c) 1sec (d) 1.8×10^{-5} sec

13. What will be the ratio of the root mean square speeds of the molecules of an ideal gas at 270K and 30K?

- a) 1:1 (b) **3:1** (c) 1:2 (d) 2:3

14. The absolute temperature of a gas is increased 4 times its original value (V_{rms}). What will be the change in r.m.s. velocity (V'_{rms}) of its molecules?

- (a) **$V'_{rms} = 2V_{rms}$** (b) $V'_{rms} = 3V_{rms}$ (c) $V'_{rms} = 4V_{rms}$ (d) $V'_{rms} = 1/2V_{rms}$

15. A gas behaves more closely as an ideal gas at

- (a) low pressure and low temperature (b) low pressure and high temperature
(c) high pressure and low temperature (d) **high pressure and high temperature**

16. The rms speed of the molecules of gas at absolute temperature T is proportional to

- (a) $\frac{1}{T}$ (b) \sqrt{T} (c) **T** (d) T^2

17. A vessel A has volume V and a vessel B has volume 2V. Both contain some water which has a constant volume. The pressure in the space above water is P_a for vessel A and P_b for vessel B.

- (a) **$P_a = P_b$** (b) $P_a = 2P_b$ (c) $P_b = 2P_a$ (d) $P_b = 4P_a$

18. Which of the following quantities is zero on an average for the molecules of an ideal gas in equilibrium?

- (a) Kinetic energy (b) **Momentum** (c) Density (d) Speed.

19. A sample of 0.177g of an ideal gas occupies 1000 cm³ at STP. The speed of the gas molecules

- (a) **1302 m/s.** (b) 1320 m/s. (c) 1300 m/s. (d) 1032 m/s..

20. The energy of a given sample of an ideal gas depends on only on its

- (a) Volume (b) pressure (c) Density (d) **temperature.**

21. The pressure of a gas kept in an isothermal container is 200 kPa. If half the gas is removed from it, the pressure will be

- (a) **100 kPa** (b) 200 kPa (c) 400 kPa (d) 800 kPa

22. The quantity $\frac{pV}{kT}$ represents

(a) mass of the gas (b) kinetic energy of the gas (c) number of moles of the gas (d) **number of molecules in the gas**

23. The rms speed of oxygen at room temperature is about 800m/s. The rms speed of hydrogen at the same temperature is about

(a) 125m/s (b) **2000m/s** (c) 8000m/s (d) 31m/s

24. There is some liquid in a closed bottle. The amount of liquid remains constant as time passes. The vapour in the remaining part

(a) **must be saturated** (b) must be unsaturated (c) may be saturated (d) there will be no vapour

25. There is some liquid in a closed bottle. The amount of liquid continuously decreasing as time passes. The vapour in the remaining part

(a) must be saturated (b) **must be unsaturated** (c) may be saturated (d) there will be no vapour

26. The average momentum of a molecule in the same for all ideal gases at the same temperature?

(a) temperature (b) number of moles (c) volume (d) **none of these**

27. Keeping the number of moles, volume and temperature the same which of the following are the same for all ideal gases?

(a) rms speed of a molecule (b) density (c) **pressure** (d) average magnitude of momentum

28. Which of the following parameters is the same for molecules of all gases of at a given temperature?

(a) mass (b) speed (c) momentum (d) **kinetic energy**

Answer-1.(a) 2.(c) 3.(b) 4.(d) 5.(a) 6.(a) 7.(c) 8.(a) 9.(c) 10.(d) 11.(b) 12.(b) 13.(b) 14.(a) 15.(d) 16.(c) 17.(a) 18.(b) 19.(a) 20.(d) 21.(a) 22.(d) 23.(b) 24.(a) 25.(b) 26.(d) 27.(c) 28.(d)

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

1. Which of the following quantities is the same for all ideal gases at the same temperature?

(a) **The kinetic energy of 1 mole** (b) The kinetic energy of 1 g (c) **The number of molecules in 1 mole** (d)) The number of molecules in 1g

2. C_v and C_p denote the molar specific heat capacities of a as at constant volume and constant pressure, respectively, then

(a) $C_p - C_v$ is larger for a diatomic ideal gas than for a monatomic ideal gas (b) $C_p + C_v$ is larger for a diatomic ideal gas than for a monatomic ideal gas (c) C_p / C_v is larger for a diatomic ideal gas than for a monatomic ideal gas (d) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monatomic ideal gas

3. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T . Assuming the gases are ideal, the correct statement(s) is(are)

(a) The average energy per mole of the gas mixture is $2kT$

(b) The ratio of speed of sound in the gas mixture that in helium gas is $\sqrt{\frac{6}{5}}$

(c) The ratio of rms speed of helium atoms to that of hydrogen molecules is $1/2$

(d) The ratio of rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$

Answers

1. (a) and (c) 2. (b) and (d) 3. (a) (b) and (d)

ANSWERS CHALLENGING TYPE QUESTIONS WITH ANSWERS

1. Write the relation between the pressure and kinetic energy per unit volume of a gas. Water solidifies into ice at 273 K . What happens to the K.E. of water molecules?

2. The absolute temperature of a gas is increased 4 times its original value. What will be the change in r.m.s. velocity of its molecules?

3. What will be the ratio of the root mean square speeds of the molecules of an ideal gas at 270 K and 30 K ?

4. A mixture of Helium and Hydrogen gas is filled in a vessel at 30°C . Compare the root mean square velocities of the molecules of these gases at this temperature.

(atomic weight of Hydrogen is 4)

A mixture of Helium and Hydrogen gas is filled in a vessel at 30°C . Compare the root mean square velocities of the molecules of these gases at this temperature.

(atomic weight of Hydrogen is 4)

ANSWERS

Ans 1:- $P = \frac{2}{3} E$. The K.E. of water molecules is partly converted into the binding energy of the ice.

Ans 2:- $V_{\text{rms}} \propto \sqrt{T}$

$$V'_{\text{rms}} \propto \sqrt{4T}$$

$$V'_{\text{rms}} / V_{\text{rms}} = 2$$

$$V'_{\text{rms}} = 2V_{\text{rms}}$$

Change in rms velocity of molecules = $V'_{\text{rms}} - V_{\text{rms}}$

$$= V_{rms}$$

$$\text{Ans 3:- } V_{rms}/V'_{rms} = \sqrt{\frac{T}{T'}} = \sqrt{\frac{270}{30}} = 3 : 1$$

$$\text{Ans 4:- } (V_{rms})_{He}/(V_{rms})_{H_2} = \{(M_{H_2})/(M_{He})\}^{1/2} = 1 : 2 \sqrt{2}$$

UNIT -X OSCILLATIONS AND WAVES

Gist and Formulae

Oscillation :-

Conditions of Simple Harmonic Motion

(a) There must be a position of **stable equilibrium**

At the stable equilibrium potential energy is minimum.

So, $dU/dy = 0$ and $d^2U/dy^2 > 0$

(b) There must be no dissipation of energy

(c) The acceleration is proportional to the displacement and opposite in direction.

That is, $a = -\omega^2 y$

Equation of SHM:-

$$(a) F = -ky \quad (b) \quad d^2y/dt^2 + \omega^2 y = 0$$

Here $\omega = \sqrt{k/m}$ (k is force constant)

Displacement (y) :-

$$y = r \sin(\omega t + \phi)$$

Condition:

When, $\phi = 0$, then, $y = r \sin \omega t$

and

When, $\phi = \pi/2$, then, $y = r \cos \omega t$

Velocity (V):- $V = dy/dt = r\omega \cos(\omega t + \phi) = v \cos(\omega t + \phi) = \omega \sqrt{r^2 - y^2}$

Here v is the linear velocity of the particle.

Condition:- When, $y = 0$, then, $V = v = r\omega$

and

When, $y = \pm r$, then, $V = 0$

$$y^2/r^2 + y^2/\omega^2 r^2 = 1$$

Acceleration (a): $a = dV/dt = (-v^2/r) \sin \omega t = -\omega^2 y$

Condition:-

When, $y = 0$, then, $a = 0$

And,

When, $y = \pm r$, then, $a = \pm \omega^2 r$.

- **Time period (T)**

(a) $T = 2\pi/\omega$

(b) $T = 2\pi\sqrt{(\text{displacement}/\text{acceleration})}$

(c) $T = 2\pi\sqrt{m/k}$

- **Frequency (f):-** It is the number of vibrations made by the body in one second.

(a) $f = 1/T$

(b) $f = 1/2\pi\sqrt{k/m}$

- **Angular frequency (ω):-**

(a) $\omega = 2\pi/T$

- **Relation between Angular frequency (ω) and Frequency (f):-** $\omega = 2\pi f = \sqrt{k/m}$

- **Energy in SHM:-**

(a) Kinetic Energy (E_k):-

$$E_k = \frac{1}{2} m\omega^2(r^2 - y^2) = \frac{1}{2} m\omega^2 r^2 \cos^2 \omega t$$

When, $y = 0$, then, $(E_k)_{\max} = \frac{1}{2} m\omega^2 r^2$ (maximum)

And

When, $y = \pm r$, then, $(E_k)_{\min} = 0$ (minimum)

(b) Potential Energy (E_p):-

$$E_p = \frac{1}{2} m\omega^2 r^2 = \frac{1}{2} m\omega^2 r^2 \sin^2 \omega t$$

$$(E_p)_{\max} = \frac{1}{2} m\omega^2 r^2$$

(c) Total Energy (E):-

$$E = E_k + E_p = \frac{1}{2} m\omega^2 r^2 = \text{conserved}$$

$$E = (E_k)_{\max} = (E_p)_{\max}$$

- **Average Kinetic Energy:-** $\langle E_k \rangle = (\frac{1}{4}) m\omega^2 r^2$
- **Average Potential Energy:-** $\langle E_p \rangle = (\frac{1}{4}) m\omega^2 r^2$
- $\langle E/2 \rangle = \langle E_k \rangle = \langle E_p \rangle$

- **Spring-mass system:-**

(a) $mg = kx_0$

(b) Time period, $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{x_0/g}$

- **Massive spring:-** Time period, $T = 2\pi\sqrt{[m + (m_s/3)]/k}$

- **Cutting a spring:-**

(a) Time period, $T' = T_0/\sqrt{n}$

(b) Frequency, $f' = \sqrt{n} f_0$

(c) Spring constant, $k' = nk$

(d) If spring is cut into two pieces of length l_1 and l_2 such that, $l_1 = nl_2$, then,

$$k_1 = (n+1/n)k,$$

$$k_2 = (n+1)k$$

and

$$k_1 l_1 = k_2 l_2$$

Spring in parallel connection:-

(a) Total spring constant, $k = k_1 + k_2$

(b) Time period, $T = 2\pi\sqrt{[m/(k_1 + k_2)]}$

(c) If $T_1 = 2\pi\sqrt{m/k_1}$ and $T_2 = 2\pi\sqrt{m/k_2}$, then,

$$T = T_1 T_2 / \sqrt{T_1^2 + T_2^2} \text{ and } \omega^2 = \omega_1^2 + \omega_2^2$$

• **Spring in series connection:-**

(a) Total spring constant, $1/k = 1/k_1 + 1/k_2$ or, $k = k_1 k_2 / k_1 + k_2$

(b) Time period, $T^2 = T_1^2 + T_2^2$

(c) $T = 2\pi \sqrt{[m(k_1 + k_2) / k_1 k_2]}$

(d) $1/\omega^2 = 1/\omega_1^2 + 1/\omega_2^2$

(e) $f = 1/2\pi \sqrt{[k_1 k_2 / m(k_1 + k_2)]}$

Law's of simple pendulum:-?

Time period of simple pendulum, $T = 2\pi \sqrt{l/g}$

(h) **Equation of motion:-** $-d^2\theta/dt^2 + (g/l)\theta = 0$

(i) **Frequency,** $f = 1/2\pi \sqrt{(g/l)}$

(j) **Angular frequency,** $\omega = \sqrt{(g/l)}$

Second Pendulum:-

(a) $T = 2$ sec

(b) $l = 0.9925$ m

Mass-less loaded spring in the horizontal alignment:-

Force, $F = -kx$

Acceleration, $a = -kx/m$

Time period, $T = 2\pi \sqrt{m/k}$

Frequency, $f = 1/2\pi \sqrt{k/m}$

Time period of mass-less loaded spring in the vertical alignment:-

$T = 2\pi \sqrt{m/k}$ and $T = 2\pi \sqrt{l/g}$

Time period of bar pendulum:-

$T = 2\pi \sqrt{I/mgl}$

$T = 2\pi \sqrt{L/g}$

Here, $L = (k^2/l) + l$

Time period of torsion pendulum:-

(a) $T = 2\pi \sqrt{I/C}$

(b) **Equation of motion:-** $-d^2\theta/dt^2 + (C/I)\theta = 0$

Here, $\theta = \theta_0 \sin(\omega t + \phi)$

(c) **Angular frequency,** $\omega = \sqrt{C/I}$

(d) **Frequency,** $f = 1/2\pi \sqrt{C/I}$

Conical Pendulum:-

Time period, $T = 2\pi \sqrt{(L \cos \theta / g)}$

Velocity, $v = \sqrt{(gR \tan \theta)}$

Restoring couple (τ):-

$$\tau = C\theta$$

Liquid contained in a U-tube:-

Time period, $T = 2\pi \sqrt{l/g}$

Electrical oscillating circuit:-

Time period, $T = 2\pi\sqrt{LC}$

Here, L is the inductance and C is the capacitance.

Angular frequency, $\omega = 1/\sqrt{LC}$

Ball in a bowl:-

Time period, $T = 2\pi\sqrt{[(R-r)/g]}$

Free vibrations:- Vibrations of a body are termed as free vibrations if it vibrates in the absence of any constraint.

Damped Vibrations:-?

Equation: $d^2y/dt^2 + 2\mu dy/dt + \omega^2 y = 0$

Here amplitude, $R = Ae^{-\mu t}$

And

$$\omega' = \sqrt{\omega^2 - \mu^2}$$

(a) $\mu \ll \omega$ signifies the body will show oscillatory behavior with gradually decreasing amplitude.

(b) $\mu \gg \omega$ signifies the amplitude may decrease from maximum to zero without showing the oscillatory behavior.

(c) In between the above two cases, the body is in the state of critically damped.

(d) Time period of oscillation, $T' = 2\pi/\omega' = 2\pi/\sqrt{\omega^2 - \mu^2}$. Thus, presence of damping factor μ in the denominator indicates an increase of time period due to damping.

Forced vibrations Equation: $d^2y/dt^2 + 2\mu dy/dt + \omega^2 y = (F_0/m) \cos pt$

Here, $\mu = r/2m$ and $\omega = \sqrt{k/m}$

Solution: $y = A \cos [pt - ?]$

Amplitude:- $A = F_0/m \sqrt{4\mu^2 p^2 + (p^2 - \omega^2)^2}$ and $A_{\max} = F_0/2\mu m \sqrt{\omega^2 - \mu^2}$.

Amplitude vibration depends upon value of $\omega = \sqrt{k/m}$. Greater the value of stiffness (k), smaller is the amplitude.

Wave

Wave Equation:- $d^2y/dt^2 = v^2 (d^2y/dx^2)$

Velocity of transverse wave, $V_t = \sqrt{T/m} = \sqrt{T/\pi r^2 \rho}$

Longitudinal wave, $V_l = \sqrt{E/\rho}$

Wave number

$$n = 1/\lambda$$

(c) **Velocity of wave:-**

$$v = f\lambda$$

Equation of progressive wave

$$y = r \sin [(\omega t \pm (2\pi/\lambda)x)]$$

$$y = r \sin (\omega t \pm kx)$$

$$y = r \sin 2\pi (t/T \pm x/\lambda)$$

$$y = r \sin 2\pi/\lambda (vt \pm x)$$

Angular wave number (k): $k = 2\pi/\lambda$

Relation between particle velocity (V) and wave velocity (v):-

$$V = (2\pi r/\lambda) v \cos[(2\pi/\lambda)(vt \pm x)]$$

$$V_{\max} = (2\pi r/\lambda) v$$

Energy transmission in a progressive wave:- $E = \frac{1}{2} m\omega^2 r^2$

Energy per unit volume:- $E = \frac{1}{2} \rho r^2 \omega^2$ Here ρ is the density of medium.

Intensity of a wave:-

$$I = 2\pi^2 \rho v f^2 r^2$$

Intensity of a wave varies directly as the square of its amplitude.

So, $I \propto r^2$

Velocity of transverse wave in stretched string:- $v = \sqrt{T/m}$, **Interference:-**

$$y_1 = a_1 \sin \omega t, y_2 = a_2 \sin(\omega t + \theta)$$

Amplitude, A = $\sqrt{a_1^2 + a_2^2 + 2 a_1 a_2 \cos \theta}$

Intensity, I = kA^2 and $I = I_1 + I_2 + 2(\sqrt{I_1 I_2}) \cos \theta$

Here, I₁ = ka_1^2 and $I_2 = ka_2^2$

Angle, θ = $\tan^{-1}[a_2 \sin \theta / (a_1 + a_2 \cos \theta)]$

Constructive interference:-

Phase difference, θ = $2n\pi$, $n = 0, 1, 2, 3, \dots$

$$A = a_1 + a_2$$

$$I_{\max} = [\sqrt{I_1} + \sqrt{I_2}]^2$$

Path difference, x = $2n(\lambda/2)$

Destructive interference:-

Phase difference, θ = $(2n+1)\pi$, $n = 0, 1, 2, 3, \dots$

$$A = 2a \cos \theta/2$$

$$I = 4a^2 k \cos^2 \theta/2$$

$$I_{\max} = 4a^2 k$$

$$I_{\min} = 0$$

Path difference, x = $(2n+1)(\lambda/2)$

Stationary Wave:-

Wave equation, y = $2a \cos(2\pi/\lambda) x \sin(2\pi/\lambda) vt$

Amplitude, A = $2a \cos(2\pi/\lambda) x$

Condition for maxima (anti-nodes), x = $k(\lambda/2)$

Condition for minima (nodes), x = $(2k+1)(\lambda/4)$

Frequency of transverse vibrations in stretched string:-

$f = (1/2l) \sqrt{T/m}$, Here l is the length, T is the tension and m is the mass.

$f = (1/D) \sqrt{(T/\pi\rho)}$, Here l is the length, T is the tension, D is the diameter and ρ is the density.

Harmonics in stretched strings:-

(a) First harmonic (fundamental frequency), $f_0 = (1/2l) \sqrt{(T/m)}$

(b) Second harmonic (first overtone), $f_1 = 2f_0 = (2/2l) \sqrt{(T/m)}$

(c) Third harmonic (second overtone), $f_2 = 3f_0 = (3/2l) \sqrt{(T/m)}$

(d) p^{th} harmonic ($p-1$ overtone), $f_{p-1} = pf_0 = (p/2l) \sqrt{(T/m)}$

Here, $p=1,2,3\dots$

Frequency of tuning fork:-

$$f \propto (l/l) \sqrt{(E/\rho)}$$

- **Phenomenon of Beats:-** If m is the number of beats per second, then, $m = f_1 - f_2$. Here f_1 and f_2 are the frequencies of the two waves.

$$y_1 = a \sin 2\pi f_1 t, y_2 = a \sin 2\pi f_2 t$$

$$y = y_1 + y_2 = A \sin 2\pi f t$$

Amplitude, $A = 2a \cos 2\pi (f_1 - f_2/2)t$, Frequency, $F = f_1 - f_2/2$

(a) **Maxima:-** $t = l/f_1 - f_2$

(b) **Minima:-** $t = 2f + 1/2(f_1 - f_2)$

Beat period (t_b):-

$$t_b = 1/f_1 - f_2$$

If m is the number of beats per second, then,

$$m = 1/\text{beat period} = f_1 - f_2$$

Loudness (L) or Intensity (I):-

$$L \propto \log I$$

$$\text{So, } L = K \log_{10}(I_1/I_0)$$

Intensity (I) and Amplitude (A):- $I \propto A^2$

Intensity (I) and distance from the source (r):- $I \propto 1/r^2$

Velocity u of longitudinal wave (sound) [Newton's Formula]:-

$$u = \sqrt{E/\rho}$$

Velocity of sound in solids:-

$$u = \sqrt{Y/\rho}$$

Velocity of sound in liquids:-

$$u = \sqrt{B/\rho}$$

- **Velocity of sound in gases:-**

$$u = \sqrt{\gamma P / \rho}$$

Here, $\gamma (=c_p/c_v)$ is the adiabatic ratio, P is the pressure and ρ is the density.

- **Various factors affecting velocity of sound:-**

(a) Effect of density:- The velocity of sound in a gas varies inversely as the square root of its density. $u_1 / u_2 = \sqrt{[\rho_2 / \rho_1]}$

(b) Effect of moisture:- $u_m / u_d = \sqrt{[\rho_d / \rho_m]}$

Since, $\rho_m < \rho_d$, then, $u_m > u_d$

This signifies sound travels faster in moist air.

(c) Effect of pressure:- $u = \sqrt{\gamma P / \rho} = \sqrt{\gamma k} = \text{constant}$

This signifies, change of pressure has no effect on the velocity of sound.

(d) Effect of temperature:- $u_t / u_0 = \sqrt{\rho_0 / \rho_t} = \sqrt{T / T_0}$

Thus, velocity of sound varies directly as the square root temperature on Kelvin's scale.

(e) Temperature coefficient of velocity of sound (α):- $\alpha = u_0 / 546 = (u_t - u_0) / t$

Overtones in open pipe:- An open pipe is open at both ends. Since air is free to vibrate at an open end, we must get an antinode at the open end.

(a) Fundamental frequency:-

Wavelength, $\lambda = 2l$

Frequency, $f = u / 2l = (1/2l) \sqrt{(\gamma P / \rho)}$

Here l is the length of the pipe and u is the velocity of sound.

(b) First overtone (Second Harmonic):-

Wavelength, $\lambda_1 = l$

Frequency, $f_1 = 2f$

(c) Second overtone (Third Harmonic):-

Wavelength, $\lambda_2 = 2l/3$

Wavelength, $f_2 = 3f$

Overtones in closed pipe:- Since air, at a closed end, is not free to vibrate, there must be a node at a closed end always.

(a) Fundamental frequency:-

Wavelength, $\lambda = 4l$

Frequency, $F = u / 4l = (1/4l) \sqrt{(\gamma P / \rho)}$

Here l is the length of the pipe and u is the velocity of sound.

(b) First overtone (Third Harmonic):-

Wavelength, $\lambda_1 = (4/3)l$

Frequency, $F_1 = 3F$

(c) Second overtone (Fifth Harmonic):-

$\lambda_2 = 4l/5$

$F_2 = 5F$

Comparison of fundamental frequencies of a closed end of an open pipe:- $f = 2F$

Doppler's Effect

(a) Source in motion, listener at rest:-

(i) Source approaching the listener:-

Modifying wave length, $\lambda' = V - a/f$

Apparent frequency, $f' = [V/V - a]f$

Change in frequency, $f = (a/V - a)f$

Here V is the velocity of sound in air and a is the velocity of source when it moves towards the listener.

(ii) Source going away from the listener:-

Apparent frequency, $f' = [V/V + a]f$

Change in frequency, $f = -(a/V + a)f$

(iii) Source crossing the listener:-

Apparent frequency of the source before crossing = $(V/V - a) f$

Apparent frequency of the source after crossing = $(V/V + a) f$

Change in frequency, $f = -(2aV/V^2 - a^2)f$

(b) Source at rest, listener in motion:-

(i) Listener moving away from source:-

Apparent frequency, $f' = [V - b/V]f$

Change in frequency, $f = (-b/V)f$

Here b is the velocity of listener.

(ii) Listener moving towards the source:-

Apparent frequency:- $f' = [V + b/V]f$

Change in frequency, $f = (+b/V)f$


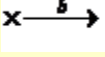

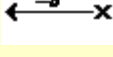
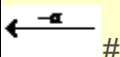
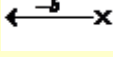
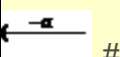
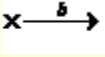
(iii) Listener crossing the source:-

Apparent frequency of the source before crossing = $(V+b/V) f$

Apparent frequency of the source after crossing = $(V-b/V) f$

Change in frequency, $\Delta f = -2fb/V$

(c) Source and listener both in the medium:- Change in frequency due to relative motion of source and listener.

Source (S) #	Listener (L) (X)	Nature of velocities	Expression for f'
		+ve, +ve	$f' = (V-b/V-a) f$
		+ve, -ve	$f' = (V+b/V-a) f$
 #		-ve, -ve	$f' = (V+b/V+a) f$
 #		-ve, +ve	$f' = (V-b/V+a) f$

(d) Effect of motion of medium:-

Apparent frequency: $f' = [(R-b)/(R-a)] f$

Here, $R = V + \omega \cos \theta$, ω is the velocity of wind, θ is the angle between direction of propagation of sound and that of wind.

Multiple choice questions with one option correct

- Q1. A cylindrical resonance tube, open at both ends, has a fundamental frequency f in air. If half of the length is dipped vertically in water, the fundamental frequency of the air column will be..
 a) $3f/2$
 b) $2f$
 c) f
 d) $f/2$
- Q2. A wave of frequency 500 Hz has a velocity of 350 m/s. The distance between two nearest points, if the wave is 60° out of phase, will be approximately.
 a) 70 cm
 b) 0.7 cm
 c) 12.0 cm
 d) 120. cm
- Q3. A wave of frequency 100Hz is sent along a string towards a fixed end. When this wave travels back, after reflection, a node is formed at a distance of 10cm from the fixed end of the string. The speeds of incident (and reflected) waves are
 a) 48 m/s
 b) 20 m/s
 c) 10/m/s
 d) 15 m/s
- Q4. A source of sound is traveling with a velocity 40km/hr towards an observer and emits sound of frequency 2000 Hz. If velocity of sound is 1220 km/hr., then the apparent frequency heard by an observer is.
 a) 1980 Hz

- b) 1950 Hz
- c) 2068 Hz
- d) 2080 Hz

Q5. A glass rod 20cm long is clamped at the middle. It is set into longitudinal vibration. If the emitted sound frequency is 4000Hz, the velocity of sound in glass will be..

- a) 2800 m/s
- b) 3200 m/s
- c) 1600 m/s
- d) 2000 m/s

Q6. A point mass m is suspended at the end of a mass-less wire of length L and cross-sectional area A . If Y is the Young's modulus of the wire, then the frequency of the oscillation for simple harmonic oscillation along the vertical direction is ..

- a) $\frac{1}{2\pi} \sqrt{\frac{LA}{mY}}$
- b) $\frac{1}{2\pi} \sqrt{\frac{LA}{mY}}$
- c) $\frac{1}{2\pi} \sqrt{\frac{YA}{mL}}$
- d) $\frac{1}{2\pi} \sqrt{\frac{mY}{AL}}$

Q7. A body is executing simple harmonic motion with an angular frequency of 2 rad/sec. The velocity of the body at 20mm displacement, when the amplitude of the motion is 60mm is

- a) 131 mm/s
- b) 188 mm/s
- c) 113 mm/s
- d) 90 mm/s

Q8. A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β , then its time period of vibration will be...

- a) $(2\pi\beta) / \alpha$
- b) β^2 / α^2
- c) α / β
- d) β^2 / α

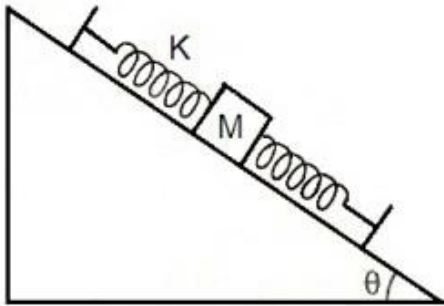
Q9. A hole is bored along the diameter of earth and a stone is dropped into the frictionless tunnel. If the radius of earth is R , then the time period of executing simple harmonic motion is ..

- a) $2\pi \sqrt{\frac{2R}{g}}$
- b) $2\pi \sqrt{\frac{R}{2g}}$
- c) $2\pi \sqrt{\frac{R}{g}}$
- d) None of these

Q10. The driver of a car traveling with speed 30 m/s towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is 330 m/s, the frequency of reflected sound as heard by the driver is ..

- a) 500Hz
- b) 550Hz
- c) 720 Hz
- d) 555 Hz

- Q11. A particle executes simple harmonic oscillation with amplitude 'a'. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is..
- T/8
 - T/12
 - T/2
 - T/4
- Q12. Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite directions when their displacement is half of the amplitude. The mean positions of the two particles lie on a straight line perpendicular to the path of the two particles. The phase difference is..
- 0
 - $2\pi / 3$
 - π
 - $\pi / 6$
- Q13. A particle is executing SHM of amplitude a with a time-period T sec. The time taken by it to move from positive extreme position to half of the amplitude is
- T/12 sec
 - 2T/12 sec
 - 3T/12 sec
 - 6T/12 sec
- Q14. On a smooth inclined plane a body of mass M is attached between two springs. The other ends of the springs are fixed to firm supports. If each spring has force constant k, the period of oscillation of the body (assume the spring are mass less) is ...



- $2\pi \sqrt{\frac{M}{2k}}$
 - $2\pi \sqrt{\frac{2M}{k}}$
 - $2\pi \left(\frac{Mg \sin \theta}{2k} \right)$
 - $2\pi \sqrt{\frac{2Mg}{k}}$
- Q15. If the spring extends by x on loading, then the energy stored by the spring is (If T is the tension in the spring) .
- $T^2 / 2x$
 - $T^2 / 2k$
 - $2k / T^2$
 - $2T^2 / k$
- Q16. The average speed of the bob of a simple pendulum oscillating with a small amplitude A and time period T as..
- 4A/T
 - $2\pi A/T$

- c) $4\pi A/T$
- d) $2A/T$

- Q17. A block of mass M is attached to the lower end of a vertical spring. The spring is hung from ceiling and has force constant value ' k '. The mass is released from the rest when the spring initially un stretched. The maximum extension produced in the length of the spring will be..
- a) $2Mg/k$
 - b) $4Mg/k$
 - c) $Mg/2k$
 - d) Mg/k
- Q18. If length of a simple pendulum is increased by 2%, then the time period ..
- a) increases by 2%
 - b) decreases by 2%
 - c) increases by 1%
 - d) decreases by 1%
- Q19. Two simple harmonic motions with the same frequency act on a particle at right angles i.e. along x axis and y axis. If the two amplitudes are equal and the phase difference is $\pi/2$, the resultant motion will be..
- a) a circle
 - b) an ellipse with the major axis along y -axis
 - c) an ellipse with the major axis along x -axis
 - d) a straight line inclined at 45° to the x -axis
- Q20. A point performs simple harmonic oscillation of period T and the equation of motion is given by $x=a \sin (\omega t + \pi /6)$. After the elapse of what fraction of the time period the velocity of the point will be equal to half of its maximum velocity
- a) $T/8$
 - b) $T/6$
 - c) $T/3$
 - d) $T/12$
- Q21. The amplitude of pendulum executing damped simple harmonic oscillation falls to $1/3$ of the original value after 100 oscillations. The amplitude fall to S times the original value after 200 oscillations, where S is ...
- a) $1/9$
 - b) $1/2$
 - c) $2/3$
 - d) $1/6$
- Q22. A hollow sphere is filled with water. It is hung by a long thread. As the water flows out of a hole at the bottom, the period of oscillation will...
- a) first increase and then decrease
 - b) first decrease and then increase
 - c) go on increasing
 - d) go on decreasing
- Q23. A body is executing S.H.M when the displacements from the mean position are 4cm and 5cm; the corresponding velocities of the body are 10cm/s and 8cm/s. Then the time period of the body is
- a) 2π sec
 - b) $\pi/2$ sec
 - c) π sec
 - d) $(3\pi /2)$ sec
- Q24. A simple harmonic oscillator has an amplitude A and time period T . The time required by it to travel from $x=A$ to $x=A/2$ is
- a) $T/6$
 - b) $T/4$

- c) $T/3$
d) $T/2$
- Q25. An open pipe of length 90cm emitting its second overtone is in unison with an open pipe emitting its third overtone. The length of the open pipe will be..
a) 1.6 m
b) 1.2 m
c) 3.2 m
d) 2.5 m
- Q26. The potential energy of a long spring when stretched by 2cm is U. If the spring is stretched by 8cm, the potential energy stored in it is...
a) 8U
b) 16U
c) $U/4$
d) 4U
- Q27. The displacement of a particle along the x-axis is given by $x = a \sin 2\omega t$. The motion of the particle corresponds to...
a) simple harmonic motion of frequency ω / π
b) simple harmonic motion of frequency $3\omega / 2\pi$
c) non simple harmonic motion
d) simple harmonic motion of frequency $\omega / 2\pi$
- Q28. Two simple pendulums of length 5m and 20m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed oscillations
a) 5
b) 1
c) 2
d) 3
- Q29. A wave has S.H.M whose period is 4 seconds while another wave which also possess SHM has its period 3 seconds. If both are combined, then the resultant wave will have the period equal to?
a) 4 Sec
b) 5 sec
c) 12 sec
d) 3 sec
- Q30. A particle starts simple harmonic motion from mean position. Its amplitude is A and time period is T. What is its displacement when speed is half of its maximum speed
a) $\frac{\sqrt{2}}{3} A$
b) $\frac{\sqrt{3}}{2} A$
c) $\frac{2}{\sqrt{3}} A$
d) $\frac{A}{\sqrt{2}}$

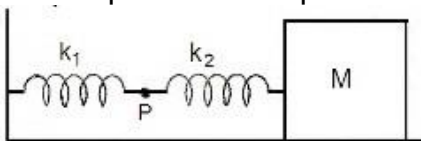
Answer

- | | | | | | | |
|--------|--------|---------|--------|---------|--------|--------|
| 1.(c) | 2 (c) | 3 (b) | 4. (c) | 5.(c) | 6. (c) | 7. (a) |
| 8. (a) | 9.(c) | 10. (c) | 11.(b) | 12. (b) | 13.(b) | 14.(a) |
| 15.(b) | 16.(a) | 17.(d) | 18.(c) | 19. (a) | 20.(d) | 21.(a) |

22. (a) 23.(c) 24.(a) 25. (b) 26. (b) 27.(a) 28.(c)
- 29.(c) 30. (b)

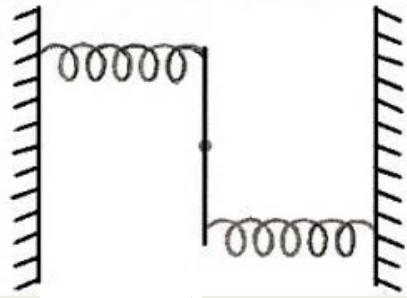
Multiple choice questions with one and more than one option correct

- Q1. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01m has a total mechanical energy of 160J. Its..
- maximum potential energy is 160J
 - maximum kinetic energy is 100J
 - minimum potential energy is zero
 - minimum potential energy is 100J
- Q2. Two simple harmonic motions acts on a particle. These harmonic motions are $x=A \cos(\omega t + \delta)$, $y=A \cos(\omega t + \alpha)$ when $\delta=\alpha + \pi/2$, the resulting motion is .
- a circle and the actual motion is clockwise
 - an ellipse and the actual motion is counterclockwise
 - an ellipse and the actual motion is clockwise
 - a circle and the actual motion is counter clockwise
- Q3. A simple pendulum performs simple harmonic motion about $x=0$ with an amplitude 'a' and time period T. The speed of the pendulum at $x=a/2$ will be..
- $\frac{\pi a}{T}$
 - $\frac{3\pi^2 a}{T}$
 - $\frac{\pi a \sqrt{3}}{T}$
 - $\frac{\pi a \sqrt{3}}{2T}$
- Q4. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



- $\frac{k_1 A}{k_2}$
- $\frac{k_2 A}{k_1}$
- $\frac{k_1 A}{k_1 + k_2}$
- $\frac{k_2 A}{k_1 + k_2}$

- Q5. A uniform rod of length and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constant k . The springs are fixed to rigid supports as shown in figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is



- a) $\frac{1}{2\pi} \sqrt{\frac{2k}{M}}$
 b) $\frac{1}{2\pi} \sqrt{\frac{4k}{M}}$
 c) $\frac{1}{2\pi} \sqrt{\frac{6k}{M}}$
 d) $\frac{1}{2\pi} \sqrt{\frac{24k}{M}}$

- Q6. Two organ pipes both closed at one end, have length l and $l + \Delta l$. Neglect end-correction. if the velocity of sound in air is V . then the number of beats per second is ..
- a) $V/4l$
 b) $V/2l$
 c) $(V/4l^2) \Delta l$
 d) $(V/2l^2) \Delta l$

- Q7. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45° , then 1
- a) the resultant amplitude is $(1 + \sqrt{2})a$
 b) the phase of the resultant motion relative to the first is 90° .
 c) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion.
 d) the resulting motion is not simple harmonic.

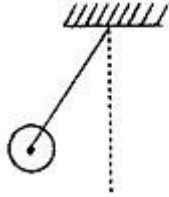
- Q8. The Function $x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$ represents SHM for which of the option(s)?
- a) for all value of A , B and C ($C \neq 0$)
 b) $A = B$, $C = 2B$ c) $A = -B$, $C = 2B$
 d) $A = B$, $C = 0$

- Q9. A student uses a simple pendulum of exactly 1m length to determine g , the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is

(are) true?

- a) Error ΔT in measuring T , the time period, is 0.05 seconds
- b) Error ΔT in measuring T , the time period, is 1 second
- c) Percentage error in the determination of g is 5%
- d) Percentage error in the determination of g is 2.5%

Q10. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (< L) is attached at its center to the free end of the rod. Consider two ways the disc is attached; (case A) the disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod – disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true ?



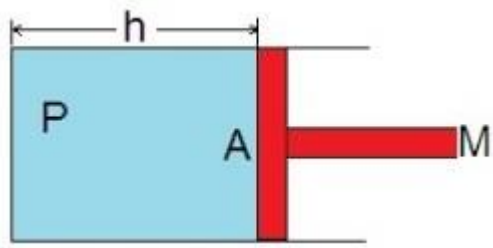
- a) Restoring torque in case A = Restoring torque in case B
- b) Restoring torque in case A < Restoring torque in case B
- c) Angular frequency for case A > Angular frequency for case B
- d) Angular frequency for case A < Angular frequency for case B

Answer

- 1. a,b
- 2. d
- 3. c
- 4. d
- 5. c
- 6. c
- 7. a,c
- 8. a,b,c
- 9. a,c
- 10. a, d

ANSWERS CHALLENGING TYPE QUESTIONS WITH.

Q1. A cylindrical piston of mass M slides smoothly inside a long cylinder at one end enclosing a certain mass of gas. the cylinder is kept with its axis horizontal. if the piston is disturbed from its equilibrium position, it oscillates simple harmonically. the period of oscillation will be



a)
$$T = 2\pi \sqrt{\frac{Mh}{PA}}$$

$$T = 2\pi \sqrt{\frac{MA}{Ph}}$$

b)

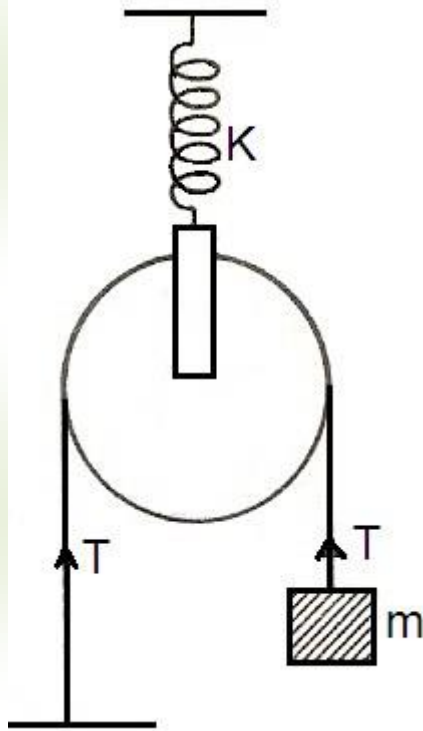
$$T = 2\pi \sqrt{\frac{M}{PAh}}$$

c)

$$T = 2\pi \sqrt{MP h A}$$

d)

Q2. A mass m is attached to a mass less inextensible string as shown in figure. the string passes over a frictionless pulley which in turn is connected to spring. If the mass is displaced from its position then the time period of oscillation of the mass m is



a) $2\pi\sqrt{(m/2k)}$

b) $2\pi\sqrt{(m/k)}$

c) $4\pi\sqrt{(m/k)}$

d) $\pi\sqrt{(m/k)}$

Q3. A body executing S.H.M when its displacement from the mean position is 4cm and 5cm, the corresponding velocity of the body is 10cm per sec and 8cm per sec. then the time period of the body is

a) 2π sec

b) $(\pi/2)$ sec

c) π sec

d) $(3\pi/2)$ sec

Q4. A particle of mass is executing oscillations about the origin on the x-axis. Its potential energy is $V(x)=k|x|^3$, where k is a positive constant. If the amplitude of oscillation is a , then its time period T is

a) proportional to $1/\sqrt{2}$

b) proportional to \sqrt{a}

c) independent $a^{3/2}$

d) none of these

Q5. If A is the area of cross-section of spring is its length E is Young's modulus of the material of the spring then time period and force constant of the spring will be respectively ..

$$a) T = 2\pi \sqrt{\frac{EA}{ML}}, k = \frac{L}{EA}$$

$$b) T = \frac{1}{2\pi} \sqrt{\frac{EA}{ML}}, k = \frac{A}{EL}$$

$$c) T = \frac{1}{2\pi} \sqrt{\frac{EL}{MA}}, k = \sqrt{\frac{EA}{L}}$$

$$d) T = 2\pi \sqrt{\frac{EL}{EA}}, k = \frac{EA}{L}$$

- Q6. The frequency of note emitted by source changes by 20% as it approaches observer. As it recedes away from him, the apparent frequency will be different from the actual frequency by
- 20%
 - 17.4%
 - 16.67%
 - 14.3%

ANSWERS

1.

When piston is moved inside then volume will decrease and pressure will increase

Let $P_1 = P =$ Initial pressure

$V_1 = V =$ Initial volume

let $P_2 = P + \Delta P =$ Final pressure

$V_2 = V - \Delta V =$ Final Volume

Using Boil's law

$$P_1 V_1 = P_2 V_2$$

OR

$$PV = (P + \Delta P)(V - \Delta V)$$

$$PV = PV + V\Delta P - P\Delta V - \Delta P\Delta V$$

Since ΔV and ΔP is very small

$$V\Delta P = P\Delta V$$

$$\Delta P = P\Delta V / V$$

Multiplying above equation by A

$$A\Delta P = (PA\Delta V) / V$$

Restoring force acting on the piston opposite to displacement due to excess pressure

$$F = A\Delta P$$

$$F = (PA\Delta V) / V$$

$\Delta V = Ax$ here x is displacement of piston

$$F = (PA^2 x) / V$$

$$F = (PA^2 / V) x$$

comparing above equation with $F = kx$ equation for SHM we get

$$k = PA^2 / V$$

Now period $T=2\pi\sqrt{m/k}$

$$T = 2\pi \sqrt{\frac{MV}{PA^2}}$$

$$T = 2\pi \sqrt{\frac{MAh}{PA^2}}$$

$$T = 2\pi \sqrt{\frac{Mh}{PA}}$$

Answer: (a)

2.

Force stretching the spring $= 2T = 2mg$. If the mass moves down by y , then the extension in the spring $= y/2$

$\therefore 2mg = k(y/2)$ acceleration $= g = (k/4g)y$

$\therefore \omega = (k/4g)$

$T = 4\pi \sqrt{m/k}$

Answer: (c)

3.

Form the formula for velocity

$$V = \omega \sqrt{a^2 - y^2}$$

$$10 = \omega \sqrt{a^2 - 4^2}$$

$$100 = \omega^2 (a^2 - 4^2)$$

$$\frac{100}{\omega^2} + 4^2 = a^2$$

and

$$8 = \omega \sqrt{a^2 - 5^2}$$

$$\frac{64}{\omega^2} + 5^2 = a^2$$

$$\frac{100}{\omega^2} + 4^2 = \frac{64}{\omega^2} + 5^2$$

$$\frac{100}{\omega^2} - \frac{64}{\omega^2} = 5^2 - 4^2$$

$$\frac{36}{\omega^2} = 9$$

$$\omega^2 = 4$$

$$\omega = 2$$

$$\frac{2\pi}{T} = 2$$

$$T = \pi$$

Answer: (c)

4.

$$V(x) = k|x|^3$$

$$\text{since } F = -dV(x) / dx = -3k|x|^2$$

comparing above equation with $F = m\omega^2 x$ we get

$$-3k|x|^2 = -m\omega^2 x$$

$$\text{Thus } \omega^2 = 3kx/m$$

here x is displacement given by $x = a \sin \omega t$

$$\omega = \sqrt{\frac{3ka \sin \omega t}{m}}$$

$$\omega \propto \sqrt{a}$$

$$T \propto \frac{1}{\sqrt{a}}$$

Answer: (a)

5.

According to the formula for Young's Modulus

$$E = FL / A\Delta L$$

Here ΔL is increase in length of spring

$$F = EA\Delta L / L$$

Now, according to Hook's law

$$F = k\Delta L$$

where k is the spring constant

From above equations

$$k\Delta L = EA\Delta L / L$$

$$k = EA/L$$

$$\text{Time period } T = 2\pi \sqrt{M/k}$$

$$T = 2\pi \sqrt{ML/EA}$$

Answer: (d)

6.

Case I source is moving towards observer Apart frequency $n' = 1.2n$ thus

$$1.2n = \left(\frac{v}{v - v_s} \right) n$$

$$1.2 = \frac{v}{v - v_s}$$

$$v_s = \frac{v}{6}$$

Case II source is moving away from observer Apparent frequency n''

$$n'' = \left(\frac{v}{v + v_s} \right) n$$

$$n'' = \left(\frac{v}{v + \frac{v}{6}} \right) n$$

$$n'' = \frac{6n}{7}$$

$$\left(\frac{n - n''}{n} \right) \times 100 = \left(\frac{n - \frac{6n}{7}}{n} \right) \times 100$$

$$\left(\frac{n - n''}{n} \right) \times 100 = \frac{100}{7} = 14.3\%$$

PHOTO GALLERY



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DURING VALEDICTORY FUNCTION**



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